

Sequential Auction Design and Participant Behavior

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To the glory of God

in

obedience, humility, and service.

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TABLE OF CONTENTS

DEDICATION	iii
ACKNOWLEDGEMENTS	iv
LIST OF TABLES	vii
LIST OF FIGURES	viii
SUMMARY	x
I INTRODUCTION	1
II LITERATURE REVIEW	4
2.1 Brief Introduction to Auction Theory	4
2.1.1 Benchmark Auction Models	4
2.1.2 Sequential Auctions	6
2.2 Internet Auctions	7
III A MODEL OF MULTI-PERIOD FORWARD AND REVERSE AUCTIONS	9
3.1 Introduction	9
3.2 Background	9
3.3 Auction Environment	10
3.4 Auction Models	14
3.4.1 Supplier's Perspective	15
3.4.2 Buyer's Perspective	18
3.5 Analysis	21
3.5.1 Equilibrium Auction Design	22
3.5.2 Comparative Statics	32
3.5.3 Expected Value of Information	33
3.6 Conclusions	38
IV KEY FACTORS IN SUCCESSFUL TIMBER PROCUREMENT AUCTIONS	40
4.1 Literature Review	40
4.1.1 Procurement Process	42

4.1.2	Adoption of Online Auctions	45
4.1.3	Overview of B2B Auction Environment	46
4.2	Characteristics of Successful Auctions	47
4.2.1	Seasonal Patterns	48
4.2.2	Description of Good	54
4.2.3	Species of Timber	56
4.2.4	Bidders	57
4.2.5	Demand and Reservation Price	58
4.3	Regression Analysis	58
4.4	Summary	59
4.4.1	Internal Factors	59
4.4.2	External Factors	63
4.4.3	Selected Pricing Rule	65
4.5	Industrial Implications	67
4.6	Recommendations for Additional Data Collection	68
4.7	Future Research	69
V	SUMMARY	71
APPENDIX A	— PROOFS OF PROPOSITIONS 3.4.1-3.4.8	74
APPENDIX B	— PROOFS OF PROPOSITIONS 3.5.1-3.5.6	97
APPENDIX C	— EQUILIBRIUM FIGURES AND TABLES	135
APPENDIX D	— COMPARATIVE STATICS	155
APPENDIX E	— EXPECTED VALUE OF INFORMATION	160
REFERENCES	187
VITA	193

LIST OF TABLES

1	Conditions for Forward and Reverse Auctions as Equilibrium Auction Preference	25
2	Conditions for Alternating Auction as Equilibrium Auction Preference . . .	25
3	Conditions when Alternating Auction is a Compromise	26
4	Model Comparison - Change in Expected Return With an Increase in the Number of Periods	33
5	Comparison of Prices and Supply in Rainy and Dry Months	53
6	Comparison of Bids for Custom and Standard Cuts of Timber	55
7	Sawlog and Plylog Supply vs. Time between auction closes	55
8	Regression Variables	59
9	Regression Model	60
10	Descriptive Statistics	60
11	Correlation Matrix	61

LIST OF FIGURES

1	Indifference Surfaces for up to 50 Periods	27
2	Indifference Surfaces for up to 50 Periods	31
3	Pulp & Paper Supply Chain	43
4	Procurement Process	44
5	RAWS station locations [US Wildfire Assessment Service Web Site]	50
6	2001 Softwood Pulpwood Production by State and Broad Species [Johnson and Steppleton, 2003]	50
7	Expected and 2004 Actual Monthly KBDI values in the Southeast	52
8	Climate Divisions in the South [US Forest Service Web Site]	53
9	Expected and 2004 Actual Monthly Rainfall in the Southeast	54
10	Scatterplot of Effect of Time between Sales on Plylog and Sawlog Online Supply	55
11	Major Forest Cover Types in the Southern United States [Eyre 1980]	57
12	2001 Hardwood Pulpwood Production by State and Broad Species [Johnson and Steppleton, 2003]	58
13	Scatterplot of Effect of Time between Sales on Standard and Custom Online Supply	65
14	Supplier FA/RFA Indifference Surface with Preference: FA above; RFA below	135
15	Supplier RA/RFA Indifference Surface with Preference: RA below; RFA above	135
16	Supplier RA/FA Indifference Surface with Preference: RA below; FA above	136
17	Mesh Supplier FA/RFA and Smooth RA/FA Indifference Surfaces	136
18	Mesh Supplier RA/RFA and Smooth RA/FA Indifference Surfaces	137
19	Buyer FA/RFA Indifference Surface with Preference: FA below; RFA above	137
20	Buyer RA/RFA Indifference Surface with Preference: RA above; RFA below	138
21	Buyer RA/FA Indifference Surface with Preference: RA above; FA below .	138
22	Mesh Buyer FA/RFA and Smooth RA/FA Indifference Surfaces	139
23	Mesh Buyer RA/RFA and Smooth RA/FA Indifference Surfaces	140
24	Three Buyer Indifference Surfaces	141
25	All Six Indifference Surfaces	141
26	Indifference Surfaces - Different View	142

27	Table of Difference between Supplier RA/FA and FA/RFA Indifference Surfaces	143
28	Tables of Difference between Supplier Indifference Surfaces	144
29	Table of Difference between Buyer Indifference Surfaces	145
30	Table of Difference between Buyer RA/RFA AND FA/RFA Indifference Surfaces	146
31	Tables of Difference between Supplier and Buyer RA/RFA and FA/RFA Indifference Surfaces	147
32	Tables of Difference between Supplier and Buyer Indifference Surfaces . . .	148
33	Tables of Difference between Supplier and Buyer Indifference Surfaces . . .	149
34	Table of Difference between Supplier RA/FA and Buyer RA/RFA Indifference Surfaces	150
35	Tables of Difference between Supplier and Buyer Indifference Surfaces . . .	151
36	Table of Expected Profit to Supplier from RA and FA Auction	152
37	Table of Expected Surplus to Buyer from RA and FA Auction	153
38	Table of the Number of Buyers for Upper and Lower Bounds of Equilibrium Region	154

SUMMARY

This thesis studies the impact of sequential auction design on participant behavior from both a theoretical and an empirical viewpoint. In the first of the two analyses, three sequential auction designs are characterized and compared based on expected profitability to the participants. The optimal bid strategy is derived as well. One of the designs, the alternating design, is a new auction design and is a blend of the other two. It assumes that the ability to bid in or initiate an auction is given to each side of the market in an alternating fashion to simulate seasonal markets. The conditions for an equilibrium auction design are derived and characteristics of the equilibrium are outlined. The primary result is that the alternating auction is a viable compromise auction design when buyers and suppliers disagree on whether to hold a sequence of forward or reverse auctions. We also found the value of information on future private value for a strategic supplier in a two-period case of the alternating and reverse auction designs.

The empirical work studies the cause of low aggregation of timber supply in reverse auctions of an online timber exchange. Unlike previous research results regarding timber auctions, which focus on offline public auctions held by the U.S. Forest Service, we study online private auctions between logging companies and mills. A limited survey of the online auction data revealed that the auctions were successful less than 50% of the time. Regression analysis is used to determine which internal and external factors to the auction affect the aggregation of timber in an effort to determine the reason that so few auctions succeeded. The analysis revealed that the number of bidders, the description of the good and the volume demanded had a significant influence on the amount of timber supplied through the online auction exchange. A plausible explanation for the low aggregation is that the exchange was better suited to check the availability for custom cuts of timber and to transact standard timber.

CHAPTER I

INTRODUCTION

Business-to-business (B2B) commerce includes several types of transactions between firms. These transactions include purchases of material, services, technology, and financial assets. When the labor to carry out these transactions is replaced with electronic data processing and Internet communication, the economic transactions are referred to as B2B e-commerce.

General transactions between firms traditionally begin with a buyer searching for inputs or a supplier searching for buyers of its goods and services. The suppliers and buyers may search for each other through advertising, brokers, dealers, trade shows, or industry associations. Once a potential partner is located, some type of negotiation process begins concerning product specifications and prices. This negotiation may conclude with either a short-term (spot) contract or the formation of a long-term contract. After the agreement has been made, the transaction involves ordering, billing, transportation arrangements, confirmation of payments and acceptance of delivery. Each of these steps toward the successful completion of a transaction can be aided by e-commerce. Internet technologies are useful in reducing search costs before the transaction, communication costs during the transaction, and recording costs after the transaction (Lucking-Reiley, 2001).

A positive rate of growth is expected for electronic commerce in the business sector with B2B e-commerce revenues expected to total \$2.7 trillion by the end of 2004 (Choices, 2003). The potential market for B2B e-commerce has been estimated to reach \$4.5 trillion worldwide by 2005 (Goldman Sachs, 2000). In recent years, the advances in Internet technology have allowed for increased use of Internet auctions for exchanging goods and services between businesses. Of 294 companies surveyed in 2003, 25% participated in an online auction during Q3, but among those, 45% increased their usage over the past quarter. Large-volume purchasers were four times more likely to use auctions than their smaller counterparts (Forrester Research, 2003).

The expected gains in productivity have been divided into four areas: automation of transactions, new market intermediaries, consolidation of supply and demand through organized exchanges, and changes in the extent of vertical integration (Lucking-Reiley, 2001). In this research, we focus on market intermediaries. Some of the new market intermediaries are auctioneers. The Internet has become a viable option for buying and selling excess inventory of commodities between businesses (Keskinocak and Tayur, 2001). Some auctioneers hold auctions of surplus inventory for suppliers (forward auctions). Other auctioneers hold reverse auctions for buyers, in which sellers compete for a procurement contract. Reverse auctions sponsored by large buyers are most common. In B2B transactions, the benefits of Internet auctions can translate into large scale savings. Freemarkets, a major online procurement exchange that assists firms and government agencies in designing reverse auctions, boasts that some of its members have seen an average annual savings of over 20% in their procurement because of their use of online reverse auctions (FreeMarkets, 2002).

Despite their popularity, as firms have begun to navigate the use of online auctions as a new tool to aid in cost reduction across the supply chain, criticisms of the reverse auction mechanism have arisen in industry and in academic research. Auctions typically used for procurement of commodity goods have been used for bidding on government and construction contracts. In the states of Minnesota and California laws were passed to forbid the use of reverse auctions to procure government construction projects (Plumbing & Mechanical, 2003).

In the automotive industry, suppliers formed a regulation group to create rules for the use of reverse auctions due to buyers using the auctions unfairly. Buyers were believed to use the auction only to see how low suppliers would bid and would not award the contract to the lowest bidder (Automotive News, 2002). Although suppliers are taking the initiative to create these rules, they must depend on buyer voluntary compliance because there is no provision for enforcement.

In research conducted on the impact of reverse auctions on buyer-supplier relationships in the automotive industry, relatively small suppliers that have participated in a single reverse auction initiated by a large buyer with whom they had an established relationship

expressed concerns over the buyer using the reverse auction opportunistically and forsaking the non-price benefits by which suppliers differentiate themselves (Jap, 2003). In the study of sequential reverse auctions, bidders have been found to self-select out of future auctions. They also respond to repeated losses by bidding less (Garvin and Kagel, 1994).

The two works that comprise this thesis investigate an alternative to the reverse auction and study an online exchange in which more than half of reverse auctions end without any bidders. The studies are linked by a growing interest in how firms can effectively use the online auction. The merging of Internet technology with the auction mechanism has lifted previous constraints on market visibility and increased strategic possibilities for transacting goods via an auction. This is true for firms in several industries, including the pulp and paper industry, which is the focus of the second investigation.

Motivated by the impact of seasonal cycles on timber prices, a sequential auction is designed in which buyers and suppliers of a commodity good can bid in or initiate auctions. Large buyers initiate reverse auctions and small suppliers initiate forward auctions. When the seasonal cycles are distinct and unpredictable and the demand fluctuates above forecasts, a supplier with needed supply is able to initiate an auction. A marketplace in which both types of auctions are exercised might have fewer concerns over fair play as both market sides are able to take power positions of auctioneer. In addition, suppliers have an incentive to become familiar with the technology needed to take advantage of online auctions when in the role of auctioneer.

In the second study, we focus on understanding the cause of a lack of success with reverse auctions in an industry exchange. In talking with industry representatives and in reviewing websites for timber sales, it appears that less than half of auctions initiated for timber receive one or more bids. Using a data set from an industry partner, we investigate possible causes for the lack of participation by considering the internal and external factors that influence the success of the auctions.

CHAPTER II

LITERATURE REVIEW

2.1 Brief Introduction to Auction Theory

2.1.1 Benchmark Auction Models

[?]. There has been much research in the area of auctions. The initial research focused on understanding and comparing the different types of auctions and determining whether one type outperformed another in terms of efficiency and profit to the single auctioneer. (An auction is deemed efficient if the item for auction is won by the bidder who values it most.) The seminal work was done by Vickrey (1961), Myerson (1981), and Riley and Samuelson (1981). They lay the framework and generalize the Revenue Equivalence Theorem, a key paradigm of auction theory.

The Revenue Equivalence Theorem asserts the equivalence of the expected revenue to the auctioneer regardless of which one of the following four auction forms is used. The four basic auction models are the English auction, the Dutch auction, the first-price sealed-bid auction, and the second-price sealed-bid auction (Vickrey auction). The English auction is commonly used online (Jap, 2002). The auctioneer starts the bidding at a reserve price and bidders signal their desire to pay successively higher prices verbally (open cry auction) or otherwise. Bidders drop out once the current price exceeds their high bid and the last remaining bidder wins the auction item and pays a price equal to the highest losing bid. In the Dutch auction, the auctioneer starts with a high bid and decreases it until a bidder signals a willingness to pay. That bidder wins and pays that price. In the sealed-bid auction, bidders simultaneously submit their bids to the auctioneer knowing that the winning bidder will pay his or her bid (first-price) or the highest losing bid (second-price).

The basic assumptions of the benchmark model (McAfee and McMillan, 1987), from which the Revenue Equivalence Theorem is derived, are as follows:

(A1) Independent Private Values: A bidder i 's valuation v_i of the object being auctioned

is his private information and does not depend on the knowledge of other bidders.

(A2) Symmetry: Bidder i assumes that any other bidder j 's valuation is an unobserved random variable $v_j, j \neq i$. The valuations are drawn from the same cumulative distribution $F(v)$, $v \in [\underline{v}, \bar{v}]$. The density function is $f(v)$.

(A3) Risk Neutrality: Bidders are risk neutral, which implies that each bidder maximizes expected return.

Some general assumptions of this model are that the seller has announced and is committed to auction rules which he or she communicates to all bidders and all bidders know the number of other bidders, their risk neutrality, and the probability distribution of valuations. Based on this information, each bidder decides how to bid. Each bidder bids an amount that is a function of his or her valuation at a Bayesian Nash equilibrium. The payment is a function of the bids alone.

This model has been thoroughly investigated over the past forty years and four interesting results have been noted. The first result is that the Dutch auction is strategically equivalent to the first-price sealed-bid auction. Each bidder will bid an amount less than his or her true valuation. How much less depends on (i) the probability distribution of the other bidder's valuations and (ii) the number of competing bidders. The second result is that the English Auction and the second-price sealed bid auction are equivalent in the sense that they both have a unique dominant strategy equilibrium, that being to bid one's true valuation. At equilibrium, the winner will be the bidder who values the object most, and the price paid will be the second highest valuation. The third result is that the winner in all four auction forms will be the bidder who values the object the most. Hence, these auction forms are Pareto optimal or efficient. For the English auction and the second-price sealed bid auction, this result holds when the information structure is asymmetric as well. The fourth result is the Revenue Equivalence Theorem mentioned previously.

McAfee and McMillan (1987) give an excellent review of the literature. Basic questions that the early research addresses are: What is the best selling mechanism from the point of view of the monopolist? Should the seller impose a reserve price and if so, what should that

price be? Should the seller require payment from all bidders? What information should the seller share with bidders about the good? These questions have been clearly answered for the four basic types of auctions for a single auction of a unique good with a monopolistic auctioneer.

2.1.2 Sequential Auctions

As noted previously, the English auction is the most commonly used auction type. Subsequent to the study of a single English auction, researchers began to study the results of employing them in a sequence to sell multiple goods. Some key differences between single and multiple-period auctions are: In a sequential auction, (1) learning occurs between bidders and between the bidder and the auctioneer; and (2) parameters such as the number of bidders, their valuations, and the quality of the good may change between periods.

Klemperer (1999) provides an extensive survey of auction theory including a section devoted to research in the area of sequential auctions. In this section, he notes that the analysis of sequential auctions is well-developed for the special case in which no buyer wants more than one unit. Specifically, for the auction of homogenous goods with risk neutral, symmetric bidders with independent private values whose payment is a function of the bids alone, revenue equivalence holds. This is true whether the goods are sold sequentially or simultaneously (Weber, 1983; Maskin and Riley, 1989; Bulow and Klemperer, 1994). Therefore, for a sequential auction in which those assumptions are made, the expected revenue to the auctioneer is the same regardless of whether an English, Dutch, first-price or second-price auction is employed. His survey does not include any work on sequential auction designs in which the role of the auction participants may change. The first study of this thesis contributes to this area by studying such an auction design.

Empirical verifications of the revenue equivalence result for sequential auctions have led to research results that give insight into the price trends over the consecutive auction rounds. Milgrom and Weber (1982) and Weber (1983) present theoretical analysis of the price trend in a forward sequential auction of identical objects and conclude that expected prices (expected revenue to the auctioneer) should remain the same throughout the sequence

of auctions. However, in the case of forward auctions and because of symmetry for reverse auctions as well, the empirical evidence shows that the actual price may either decline or rise sequentially. During a forward auction, in which the price ascends over the course of the auction, the evidence that the winning price in each forward auction of a sequence actually declines is termed the "declining price anomaly." Much of the research on non-Internet sequential auctions focuses on the observance of this anomaly and its theoretical explanations. See Ashenfelter (1989), McAfee and Vincent (1993), Von de Fehr (1994) and Menezes and Montiero (1999) for plausible explanations. The data used in the empirical analysis of these non-Internet sequential auctions was collected from auctions for a variety of items including wool, stamps, condominiums, cable television licenses and wine (Menezes and Montiero, 1999). Some of these items were sold in business-to-consumer (B2C) markets while others were sold in B2B markets. Apart from studies of price trends, this thesis contributes to the sequential auction literature by specifically considering the English auction design in which the roles of auction participants may change.

2.2 Internet Auctions

In the area of online auctions, much work has focused on transactions between businesses and consumers. Researchers have conducted studies of transactions on the popular eBay website (Lucking-Reiley et al., 2000; Bapna, Goes and Gupta, 2003). Aside from eBay, Beam and Segev (1998) considered 100 various auction sites, 98 of which were B2C, to identify defining characteristics of online auctions and determine the major differences from non-Internet auctions. They cite that the online auctions are typically English auctions that close approximately every week with the price determined by bids alone. They acknowledge the opportunities for further research into B2B e-commerce and the empirical work in the second study of this thesis expands the body of research in this field.

The common assumptions made in the literature on how bidders enter and exit an auction are not typical for Internet auctions. There are many theoretical developments that assume the clock model of the English auction as defined by Milgrom and Weber (1982) as their basis. In this model, there are two clocks; an increasing price clock and a

decreasing "active-bidders" clock. Each bidder holds up a card (or presses a button or uses some alternative device) to signal his or her willingness to buy at the current price. As the price rises on the price clock, the number on the "active-bidders" clock decreases as some of the bidders lower their cards and forego, irrevocably, the possibility of buying the unit currently on sale. If k units are for sale, the price clock stops rising when the number on the active-bidders clock equals k . The k bidders each receive one unit at the current price on the price clock.

Typical Internet auctions can differ from this model in that any bidder with access to the auction web site may increase the price until the time clock runs out. The auctioneer does not increase the price but typically, in B2B auctions, only opens the auction to an invited group. The invited group of bidders are qualified to sell items or purchase the item for sale. One impact of this change in the identifying assumption is that the number of bidders in an auction is exogenously determined. Therefore, it is possible that the total number of bidders is not known *ex ante*. Even when bidders are members of an exchange and accept an invitation to bid, the number of bidders that will actually submit a bid to buy a unit is not known until after the auction has ended (Millet et al., 2004). In the second study, we consider a plausible explanation for auctions that end with no bidders in an online exchange for timber. It is with the motivation of online B2B auctions that we consider the next two works.

CHAPTER III

A MODEL OF MULTI-PERIOD FORWARD AND REVERSE AUCTIONS

3.1 Introduction

Industries with supply or demand influenced by seasonal cycles or weather conditions, such as the pulp and paper industry (Haynes 1998) and the electricity industry (Burger 2004) hold spot market auctions that may allow temporary shifts in auction-market roles. For example, a supplier with excess timber to sell during a rainy, season-induced supply shortage can potentially sell it at a premium. The wet weather that causes supply shortages in timber gives few suppliers with excess timber the opportunity to initiate an auction in a market where they usually act as bidders, but there is little evidence that they take advantage of this opportunity. We compare the expected profit to the supplier and the expected surplus of the buyer when each participates in a sequential auction as a bidder, as an auction-initiator, or alternates between both roles. When there are numerous suppliers and few buyers, our models characterize the equilibrium conditions under which both suppliers and buyers prefer each auction design and under which neither market side agrees on the auction design. In these cases, we consider a new auction design, the alternating auction, as a second option for market equilibrium. In addition, we characterize the long term behavior of the expected profit from each auction design as the number of periods increase. We also determine the value to a strategic supplier of learning her future private value before the auction sequence commences.

3.2 Background

The Internet introduces the capability of businesses to aggregate suppliers or buyers and easily communicate their need to buy or sell excess inventory. In addition, it is much easier for both forward and reverse auctions to exist for the same commodity because the set up

costs for the auction-initiator have virtually been eliminated (Lucking-Reiley, 2001). Several auction intermediaries have created websites. Many of the websites are industry specific (e.g., ForestExpress, Covisint, and ChemConnect). These websites were created by groups of companies interested in procuring materials through online exchanges that host auctions. The extent to which B2B exchanges have been integrated with overall sourcing strategies has varied significantly across industries (Berryman and Heck, 2001; Grey et al., 2002)

Capacity-intensive industries with supply or demand influenced by seasonal cycles or weather conditions, such as the pulp and paper industry (Haynes, 1998) and the electricity industry (Burger, 2004), use B2B exchanges to hold spot market auctions. The cyclic nature of procurement in such industries may influence the roles of auction participants who are able to behave strategically. We are unaware of any auction models that account for the possibility that auction roles may change at points in time.

We study three issues in this chapter. First, we study the expected profit and surplus for participating in sequential auctions of each type: forward, reverse, and alternating. Second, we determine the conditions for each auction design to serve as a market equilibrium and characterize the behavior of the equilibrium with changes in the number of periods. Third, we determine the expected value of information on the future private value for a strategic supplier bidding in the reverse or alternating auction design.

In Chapter 3.3, we present the auction environment for multiple periods. In Chapter 3.4, we develop the auction models and present the expected return and bid functions. In Chapter 3.5, we develop and characterize the equilibrium auction design and conduct further analysis. In Chapter 3.6, we summarize the results, make concluding remarks and discuss extensions of this work.

3.3 Auction Environment

Auctions are often used when there are asymmetries of information and a monopolistic or monopsonistic (i.e., supply-side monopoly) party. The type of auction mechanism that is most fitting for a given market depends on the number of market participants. Forward auctions are used primarily when there is one supplier and there are many buyers who bid the

price up. They are typically used to sell slow-moving or excess inventory. Reverse auctions are used primarily when there is one buyer and many suppliers who bid the price down (Cassady, 1967). Reverse auctions are best implemented in the B2B context when there is competition among bidders, when the purchase price of the good for auction constitutes the largest portion of its value, and when the auctioning firm has carefully considered its overall sourcing strategy (Jap, 2002).

For an auction assuming homogenous goods with risk neutral, symmetric bidders with independent private values whose payment is a function of the bids alone, revenue equivalence holds. This is true whether the goods are sold sequentially or simultaneously (Weber, 1983; Maskin and Riley, 1989; and Bulow and Klemperer, 1994). An auction model with the above assumptions is referred to as the independent private values (IPV) model (Weber, 1983). For a sequential IPV auction, the expected revenue to the auctioneer is the same regardless of whether an English, Dutch, first-price or second-price sealed-bid auction is employed. For our purposes, we examine the sequential English auction format, which is commonly used online (Jap, 2002).

Our auction models have three characteristics. First, we restrict our analysis to spot market (SM) auctions, which are auctions of commodity goods that are sold for cash and delivered immediately. Because the goods are sold for cash, they tend to be small in volume. There is low risk involved in procuring raw materials via spot market auctions due to lower-volume transactions of standard goods. The immediate delivery of the good indicates that there need not be a long-term relationship between the auction participants. Spot market auctions can occur more frequently because of the speed of the transaction and can therefore be modeled as sequential auctions.

To model spot market auctions as sequential auctions, we assume the IPV model and include two additional assumptions. First, we assume that the supplier and buyer valuations change randomly each period and that neither learns her or his valuation for a period until the beginning of that period. This change in private valuation is due to the change in spot supply and inventory over time. In the IPV model, the bidder marginal value for identical goods only changes as the bidder acquires multiple goods but the case of the

bidder valuation changing with time for a single unit of a set of identical goods has not been considered elsewhere.

Second, we assume that both buyers and suppliers of the commodity good are able to initiate an auction at different times in a set of sequential auctions. We further assume that any buyer has a demand equal to the number of auction periods thus desiring one unit each period. Likewise, we assume that each supplier has a capacity constraint and is only able to supply one unit for the duration of the sequence. Consequently, there must be more suppliers than periods in the auction sequence.

In the literature, there has been study of forward or reverse auctions that may be first-price in the first stage and English in the next stage. These are known as hybrid auctions (Dutra and Menezes, 2002). There has also been study of auctions in which bidders choose their level of participation (Levin and Smith, 1994; Menezes and Montiero, 2000). In each of the cases studied, the bidder chooses to either enter (and bid) or not enter in each period of a sequential auction. The choice is made based on delay or participation costs. However, they do not have the option to initiate an auction during the periods that they do not enter the auction to bid. We consider this option which has previously not been explored in the literature (Klemperer, 1999).

Although the demand and supply of the buyer and supplier, respectively, will be one unit for each auction in which they are active, the private valuation and the role of the buyer and supplier change each period so we refer to these assumptions as period-dependent (PD). The SM auction model that has these assumptions for the market participants and period-dependent valuations will be referred to as a period-dependent spot market (PDSM) auction model. Although we do not consider complementary sourcing strategies of buyers in our models, there is work on integrating short-term spot market transactions and long-term contracts (Araman et al., 2001; Cohen and Agrarwal, 1999; and Kleindorfer and Wu, 2003).

We compare the expected return of three sequential PDSM English auction models. The first is a reverse auction model (RA), the second is a forward auction model (FA), and the third combines elements of both the reverse and forward auction model (RFA). We introduce the formal models of the multi-period auctions by summarizing their important

features.

There are $B \geq 2$ large buyers indexed $j = 1 \dots B$ and $S \geq 3$ small suppliers indexed $i = 1 \dots S$. (For $B = 1$ or $S \leq 2$, one or more periods of our multi-period auction would have no competition.) At the beginning of each sequence, each buyer j has a value w_N^j , known only to buyer j , for one unit in the first auction of an N -period sequence, where N is interpreted as the number of auctions remaining. Likewise, each supplier i has a value v_N^i , known only to supplier i , for one unit in the first auction of a sequence of N auctions. Values in future periods are independent and identically distributed uniformly with cumulative distribution F , which is assumed to have a continuous density f , and f has support $[0, 1]$. The distribution of the values in each auction is known but the actual value is private. The values in future periods change as the supply of or demand for the spot market quantity changes independently for each market participant. For example, current inventory decisions and future weather patterns can affect the private value timber companies have for a given volume of timber in varying degrees.

A reverse PDSM auction is held in each period of the RA sequence and a forward PDSM auction is held in each period of the FA sequence. For the RFA model, instead of allowing a buyer or supplier the opportunity to initiate an auction at any time, we simplify the model by assuming they must alternate roles. In each period of the sequence, an eligible auctioneer (a buyer in a reverse auction period or a supplier in a forward auction period) initiates an auction for a commodity good and invites all eligible bidders to bid while the remaining eligible auctioneers wait for the next period in the sequence to participate. All eligible auctioneers have an equal likelihood of acting as the auctioneer. Before the auction begins, all eligible bidders (buyers in a forward auction period or suppliers in a reverse auction period) discover their private valuations and submit a bid b_N in the N^{th} -to-last period of the sequence. Their private valuations depend on seasonal patterns, current inventory, and the transportation costs to deliver the good to the auctioneer. After a winner is determined (based on the highest bid in a forward auction or the lowest bid in a reverse auction), one supplier exits due to supply constraints. Whether a supplier sells the good as an auctioneer or, as a bidder, wins the right to sell it, she exits the auction sequence after

participating in one transaction. Buyers do not exit the auction sequence because they have an unlimited capacity for the small volume that is transacted in the spot market. To analyze the alternating auction model, we fix the last period as a forward auction.

In the PDSM marketplace, with large buyers and small suppliers, the supplier is able to initiate an auction when there is a season-induced supply shortage and the supplier has excess supply to sell. For simplification of the model, we assume that only one random supplier has excess supply to sell. Buyers submit bids in a forward auction because they have constant output requirements, seasonal conditions reduce their input levels, and the operational risk is low when purchasing small volumes. Each buyer is one of the buyers to which all suppliers supply the commodity good.

The buyer's surplus is defined as the difference between the amount that a buyer has budgeted to pay for the good and the amount he actually pays. The supplier's profit is defined as the difference between the amount that the good costs the supplier and the amount for which she sells it. (For convenience, we use male pronouns to represent buyers and female pronouns to represent suppliers.) We assume the inventory cost for the good is negligible compared to the price of the good. Both forward and reverse auctions are assumed to be English auctions. The payment made by the buyer who initiates the auction and received by the winning supplier is equal to the second lowest bid across suppliers. Likewise, when the supplier initiates, the payment made by the winning buyer is equal to the second highest bid across buyers.

We next develop each auction model and determine the optimal bidding strategies. We prove each strategy's optimality and expected profit by mathematical induction. All proofs can be found in Appendix A.

3.4 Auction Models

In this chapter, we compare the expected profit and surplus for PDSM auction sequences of N reverse auctions, N forward auctions, and N alternating auctions. The comparison is made from the perspective of both the supplier and the buyer.

3.4.1 Supplier's Perspective

3.4.1.1 Reverse Auction

In the sequential second-price reverse auction, the bidders have the dominant strategy to bid their true valuation in the last period (Milgrom and Weber, 2000). There is a bidding function, b_N , which is the pure strategy symmetric equilibrium in the first period of a sequence having N periods. The assumption of bidder symmetry in the IPV model allows us to fix a supplier i and remove the superscript of the private valuation. As each auction ends, the value of N decrements by one and both the bid strategy and the valuation change with each new period. As the number of remaining periods N decreases, a supplier's bid strategy will change. In general, we define a supplier's bid strategy in the N^{th} -to-last period as $b_N(v_N)$, where v_N is the supplier's value of the good being auctioned in the N^{th} -to-last period. Although the function b_N is a function of several variables, we highlight its dependence on the private valuation because that is the variable that is private to the bidder and may or may not be reported as the bidder's true type for strategic purposes. (Note that by our assumption, v_N is unknown until the N^{th} -to-last period begins; therefore, we use the value distribution f defined previously in our expected profit calculations.)

The set of the bid functions for the supplier reveals that her optimal bid strategy is to inflate her bid while the set of bid functions for the buyer reveals that his optimal strategy is to bid his valuation. Because the bidders are symmetric, these are the Bayesian Nash equilibrium bid functions.

Proposition 3.4.1 *(RA Expected Supplier Profit and Bid Strategy) The optimal equilibrium bid function when competing in the N^{th} -to-last period of a PDSM reverse auction with S suppliers is*

$$b_N = v_N + \frac{N - 1}{S(S - N + 1)}.$$

Proof: See Appendix A.

The expected profit to a supplier competing in the N^{th} -to-last period of a PDSM reverse

auction with S suppliers competing is

$$P_N = \frac{(1 - v_N)^S}{S} + \frac{N - 1}{S(S - N + 1)}. \quad (1)$$

As in (1), the first term of the expected profit can be split into $\frac{1-v_N}{S}(1 - v_N)^{S-1}$, which is a part of the profit to the winning supplier with valuation v_N multiplied by the probability that v_N is less than the lowest valuation of the remaining $S - 1$ suppliers. The second term $\frac{N-1}{S(S-N+1)}$ is the amount by which the bidder with the next lowest valuation inflates her bid according to the optimal bid strategy. It is the sum of the expected profit of winning a future reverse auction and can be expanded to $\frac{1}{S(S-1)} + \frac{1}{(S-1)(S-2)} + \dots + \frac{1}{(S-N)(S-N+1)}$. This term is the other part of the profit to the winning supplier. The expected profit for an arbitrary supplier is composed of these two parts of profit to the winning supplier and explained further in Appendix A. Because the number of remaining periods is N , the number of suppliers, $S > N$.

The bid function is the private valuation of the bidder in the first period of an N -period auction sequence inflated by a function of the number on suppliers, buyers, and periods. Because the bidders are symmetric, this is the Bayesian Nash equilibrium bid function.

3.4.1.2 Alternating Auction

We now turn to the alternating auction model for comparison. When the number of periods is even, the RFA model begins with a reverse auction. In the following proposition, we see that the RFA model of alternating auctions resembles the RA model when there is an even number of periods.

Proposition 3.4.2 (*RFA Expected Supplier Profit and Bid Strategy*) *The expected profit to a supplier in the N^{th} -to-last period of a PDSM alternating auction sequence when N is even is*

$$P_N = \frac{(1 - v_N)^S}{S} + \frac{B - 3}{2(B + 1)(S - 1)} + \Delta_N, \quad (2)$$

where

$$\Delta_N = \sum_{p=1}^{\frac{N-2}{2}} \left(\prod_{q=1}^p \left(1 - \frac{1}{S - 2q + 1} \right) \right) * \left[\frac{1}{(S - 2p)(S - 2p + 1)} + \frac{B - 3}{2(B + 1)} * \frac{1}{S - 2p - 1} \right].$$

The optimal bid strategy is

$$b_N(v_N) = v_N + \frac{B-3}{2(B+1)(S-1)} + \Delta_N.$$

where Δ_N , $N > 2$, is the expected profit in subsequent auctions.

Proof: See Appendix A.

When we compare the expected profit functions in (1) and (2) we notice that the first terms are identical but the remaining terms differ considerably. The future expected profit of the alternating auction that has an even number of periods is comprised of the probability of not being selected to initiate in prior period, the probability of being selected to initiate in an upcoming forward auction, and the expected profit of winning a future reverse auction given that the supplier doesn't know her future private valuations. A detailed interpretation of this proposition is as follows.

The first term, $\frac{(1-v_N)^S}{S}$ is the expected profit of the reverse auction in the N^{th} -to-last period. The second term, $\frac{B-3}{2(B+1)(S-1)}$ is equivalent to the term $\frac{\frac{B-1}{B+1} - \frac{1}{2}}{\frac{1}{S-1}}$. Here, $\frac{1}{S-1}$ is the probability that the supplier is able to initiate a forward auction. The term $\frac{B-1}{B+1}$ is the expected price of a forward English auction when valuations are uniformly distributed between 0 and 1. The term $\frac{1}{2}$ is the expected future private valuation of the supplier. The sum-product term has three parts. The product term is the probability that the supplier is not able to initiate in any of the first p periods. The term $\frac{1}{(S-2p)(S-2p+1)}$ is the expected profit to the supplier who wins a reverse auction. This term is identical to the pattern of the future expected profit of a reverse auction in the RA model. The last part of the sum-product term is the expected profit if the supplier is able to initiate a forward auction after $2p+1$ periods have passed.

3.4.1.3 Forward Auction Model

We now turn to the forward auction sequence. In the FA model buyers compete to win excess supply from the supplier.

Proposition 3.4.3 (FA Expected Supplier Profit) *The expected profit to a supplier in the N^{th} -to-last period of a PDSM forward auction sequence with B buyers and S suppliers is*

$$P_N = \frac{\frac{B-1}{B+1} - v_N}{S} + \sum_{p=0}^{N-2} \left(\prod_{q=0}^p \left(1 - \frac{1}{S-q} \right) \right) \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S-p-1}. \quad (3)$$

Proof: See Appendix A.

The interpretation of this proposition is that the first term, $\frac{\frac{B-1}{B+1} - v_N}{S}$ is the expected profit from winning in the first period. The sum-product term is the future expected profit if the supplier is not able to host the forward auction in the first p periods but is able to host an auction in one of the remaining periods.

3.4.1.4 Alternating Auction

We now turn to the alternating auction model for comparison. When the number of periods is odd, the RFA model begins with a forward auction. In the following proposition, we see that the RFA model of alternating auctions resembles the FA model when there is an even number of periods.

Proposition 3.4.4 (*RFA Expected Supplier Profit and Bid Strategy*) *The expected profit to a supplier in the N^{th} -to-last period of a PDSM alternating auction sequence when N is odd is*

$$P_N = \frac{\frac{B-1}{B+1} - v_N}{S} + \Upsilon_N \quad (4)$$

$$\text{where } \Upsilon_N = \sum_{p=0}^{\frac{N-3}{2}} \left(\prod_{q=0}^p \left(1 - \frac{1}{S-2q} \right) \right) * \left[\frac{1}{(S-2p)(S-2p-1)} + \frac{B-3}{2(B+1)} * \frac{1}{S-2p-2} \right].$$

Proof: See Appendix A.

Note that Υ_N is the future expected profit.

The first term of both (3) and (4) are identical. The sum-product term is identical to that of (2) except that it accounts for an odd value of N . It is the probability of not being able to initiate a forward auction and the profit that the supplier receives from either initiating a future forward auction or winning a reverse auction.

3.4.2 Buyer's Perspective

In this chapter, we compare the expected surplus to the buyer in the reverse, forward, and alternating auction models. The expected surplus to a buyer participating in the N^{th} -to-last auction period will be denoted by the function, D_N .

3.4.2.1 Reverse Auction

Proposition 3.4.5 (*RA Expected Buyer Surplus*) *The expected surplus to a buyer in the N^{th} -to-last period of a PDSM reverse auction sequence with B buyers and S suppliers is*

$$\begin{aligned} D_N &= \frac{w_N - \frac{2}{S+1}}{B} + \sum_{p=1}^{N-1} \frac{\frac{1}{2} - \frac{2}{S-p+1}}{B}, \text{ if } N > 1 \\ &= \frac{w_N - \frac{2}{S+1}}{B}, \text{ if } N = 1. \end{aligned} \quad (5)$$

Proof: See Appendix A.

The total expected profit has two parts. The first term, $\frac{w_N - \frac{2}{S+1}}{B}$, is the expected surplus from being able to initiate in the N^{th} -to-last period. It is the difference between the buyer's valuation and the winning bid price, which is the average expected price of a reverse auction when the bidder valuations are uniformly distributed between 0 and 1. When there is only one period, the summation term is not applicable. Because we assume that the buyer can host more than one reverse auction, the second term is the expected profit if the buyer is able to win one or more subsequent periods when his future valuations are unknown.

3.4.2.2 Alternating Auction

Proposition 3.4.6 (*RFA Expected Buyer Surplus and Bid Strategy*) *The expected surplus to a buyer in the N^{th} -to-last period of a PDSM alternating sequence when N is an even number is*

$$\begin{aligned} D_N &= \frac{w_N - \frac{2}{S+1}}{B} + \frac{N}{2B(B+1)} \\ &\quad + \sum_{p=1}^{\frac{N-2}{2}} \frac{\frac{1}{2} - \frac{2}{S-2p+1}}{B}. \end{aligned} \quad (6)$$

The optimal bid function of the competing buyer is

$$b_N(w_N) = w_N.$$

Proof: See Appendix A.

A comparison of the expected surplus functions in (5) and (6) shows that the first terms are identical. In the second term of (6), $\frac{N}{2}$ represents the half of the N auctions that will be forward auctions, in which the buyer bids. The fraction $\frac{1}{B(B+1)}$ is the future expected surplus of the buyer who wins in the reverse auctions given that he does not know his

future private valuations. The second term of (5) and the third term of (6) are similar and account for the remaining reverse auctions that the buyer may be able to initiate given that his future private valuations are unknown. When $N = 2$, the third term in (6) equals zero (because there are no other reverse auctions in the sequence).

3.4.2.3 Forward Auction

Proposition 3.4.7 (*FA Expected Buyer Surplus and Bid Strategy*) *The expected surplus to a buyer competing in the N^{th} -to-last period of a PDSM forward auction sequence is*

$$D_N = \frac{(w_N)^B}{B} + \frac{N-1}{B(B+1)}, \quad (7)$$

and the optimal equilibrium bid strategy is

$$b_N(w_N) = w_N.$$

Proof: See Appendix A.

The first term, $\frac{w_N^B}{B}$, is the expected surplus from winning in the first period. It can be broken into two parts. The first part, $\frac{w_N}{B}$, is the profit to a buyer with valuation w_N . The second part, w_N^{B-1} , is the probability that the highest valuation of the other $B-1$ buyers is less than w_N . The second term, $\frac{N-1}{B(B+1)}$, is the sum of the expected surplus from winning $N-1$ forward auctions.

The equation for the optimal equilibrium bid function, b_N , states that the buyer's optimal bid strategy is to bid his true valuation for the good in each period. The intuition is that we assume that each buyer is able to transact in each period, so each auction that passes without a buyer winning is one less good out of N that the buyer is not able to purchase. In other words, there is no opportunity cost for winning in the current period instead of in a future period. The buyer has virtually no limit for spot market inventory and is able to win each auction. Hence the buyer will bid truthfully to acquire as many of the N goods as possible.

3.4.2.4 Alternating Auction

Proposition 3.4.8 (*RFA Expected Buyer Surplus and Bid Strategy*) *The expected surplus to a buyer in the N^{th} -to-last period of a PDSM alternating sequence when N is odd is*

$$D_N(w_N) = \frac{w_N^B}{B} + \frac{N-1}{2B(B+1)} + \sum_{p=0}^{\frac{N-3}{2}} \frac{\frac{1}{2} - \frac{2}{S-2p}}{B} \quad (8)$$

and $b_N(w_N) = w_N$.

Proof: See Appendix A.

The first term of (8) is identical to the first term of (7). The second terms are similar but the second term in (7) represents the expected surplus in only half of the remaining auctions thus requiring the third term in (7) to account for the expected surplus from future reverse auctions.

We note that the optimal bid strategy for each supplier bidding in a PDSM auction with $N > 1$ is to inflate her bid above her private valuation. The optimal bid strategy of the buyer is to bid his valuation in each period.

3.5 Analysis

We compare the performance of the alternating auction design to that of the reverse and forward auction designs. First, we derive the conditions under which each auction design is an equilibrium and when neither the reverse nor forward is an equilibrium. We then look at whether for these instances, the alternating auction design is a next best alternative. We also find the conditions under which the preferences of auction design change with changes in the number of suppliers, buyers and periods in a test range. Second, we look at how the expected profit and surplus functions change with a local increase in the number of periods. Lastly, because we assume that the private valuations change randomly each period, we will measure how much a strategic supplier should pay to learn her private value in the last period of a two-period sequence both before and after learning her private valuation in the first period.

3.5.1 Equilibrium Auction Design

To determine the equilibrium auction design, we compare the expected return from each auction design. For each side of the market, the comparison is expressed by inequalities. Three inequalities are needed to characterize the preference of each market side among the three auction designs. Each inequality is stated with respect to the number of buyers, B . When the number of buyers is above the expression, one auction design is preferred. When it is below the expression, the other design is preferred. An equilibrium auction design, if one exists, is the design that both suppliers and buyers prefer when B is within a given range.

Before the decision on the auction type between buyer and supplier is made, the private valuation is not known to either buyer or supplier. This is because a portion of the valuation of either party depends on the transportation or delivery cost of the good and therefore depends on who will initiate the auction. To account for this we integrate the expected profit and surplus functions with respect to the private valuations of the suppliers and buyers, respectively. These valuations are uniformly distributed between zero and one. The derivation of the inequalities is located in Appendix B.

Buyer Preference

In this section, we compare the N -period expected surplus to the buyer under each auction sequence.

Proposition 3.5.1 (*Buyer Reverse Auction vs. Forward Auction*) *A surplus-maximizing buyer will be indifferent between a reverse auction and a forward auction when $B = \frac{N}{\lambda} - 1$, where $\lambda = \sum_{p=0}^{N-1} (\frac{1}{2} - \frac{2}{S-p+1})$.*

Under the standard assumptions that $S \geq N + 1$ and $N \geq 2$, when $S \geq 5$ or both $S \geq 4$ and $N = 2$, the buyer will prefer the reverse auction if and only if $B > \frac{N}{\lambda} - 1$. When $S = N + 1$ for $N \in \{2, 3\}$, the buyer will never prefer the reverse auction (because no B exists for which $B < \frac{N}{\lambda} - 1$).

Proof: See Appendix B.

Proposition 3.5.2 (*Buyer Reverse Auction vs. Alternating Auction*) *A surplus-maximizing*

buyer will be indifferent between a sequence of reverse auctions and a sequence of alternating auctions when $B = \frac{N}{2(\lambda-\rho)} - 1$, where

$$\lambda = \sum_{p=0}^{N-1} \frac{1}{2} - \frac{2}{S-p+1}$$

and

$$\rho = \begin{cases} \sum_{p=0}^{\frac{N-2}{2}} \frac{1}{2} - \frac{2}{S-2p+1}, & \text{when } N \text{ is even} \\ \sum_{p=0}^{\frac{N-3}{2}} \frac{1}{2} - \frac{2}{S-2p}, & \text{when } N \text{ is odd.} \end{cases}$$

Under the standard assumptions that $S \geq N + 1$ and $N \geq 2$, when $S \geq 6$ or when both $S = 5$ and $N = 2$ or $N = 3$, the buyer will prefer the reverse auction if and only if $B > \frac{N}{2(\lambda-\rho)} - 1$. When $S = N + 1$ for $N \in \{2, 3, 4\}$, the buyer will never prefer the reverse auction (because no B exists for which $B < \frac{N}{2(\lambda-\rho)} - 1$). When $S = 4$ and $N = 2$ (the only remaining case), the buyer again will never prefer the reverse auction.

Proof: See Appendix B.

Proposition 3.5.3 (*Buyer Forward Auction vs. Alternating Auction*) A surplus-maximizing buyer will be indifferent between a sequence of forward auctions and a sequence of alternating auctions when $B = \frac{N}{2*\rho} - 1$ where ρ is as defined in Proposition 3.5.3.

Under the standard assumptions that $S \geq N + 1$ and $N \geq 2$, when $S \geq 5$ or when both $S = 4$ and $N = 2$, the buyer will prefer the forward auction if and only if $B < \frac{N}{2\rho} - 1$. When $S = N + 1$ for $N \in \{2, 3\}$, the buyer will prefer the forward auction because for any B , $\frac{N}{2(B+1)} > \rho$.

Proof: See Appendix B.

Supplier Preference

In this section, we compare the N -period expected profit to the supplier under each auction sequence.

Proposition 3.5.4 (*Supplier Reverse Auction vs. Forward Auction*) A profit-maximizing supplier will be indifferent between a reverse auction and a forward auction when

$$B = \frac{1+\gamma}{1-\gamma},$$

where

$$\gamma = \frac{2SN}{(S+1)(S-N+1)(1+\alpha S)} + 1,$$

and where

$$\alpha = \sum_{p=0}^{N-2} \left(\prod_{q=0}^p \left(1 - \frac{1}{S-q} \right) \right) \frac{1}{S-p-1},$$

Under the standard assumptions that $S \geq N+1$ and $N \geq 2$, we have that the buyer will prefer the reverse auction to the forward auction if and only if $B > \frac{1+\gamma}{1-\gamma}$.

Proof: See Appendix B.

Proposition 3.5.5 (*Supplier Reverse Auction vs. Alternating Auction*) A profit-maximizing supplier will be indifferent between a reverse auction and an alternating auction when

$$B = \frac{1+\varrho}{1-\varrho}$$

where

$$\varrho = \frac{\frac{N-1}{S(S-N+1)}(S-1) - \kappa(S-1) + \frac{1}{2} + \frac{1}{2}\tau(S-1)}{1 + \tau(S-1)}$$

and where

$$\tau = \begin{cases} \sum_{p=1}^{\frac{N-2}{2}} \left(\prod_{q=1}^p \left(1 - \frac{1}{S-2q+1} \right) \right) \frac{1}{S-2p-1}, & \text{when } N \text{ is even} \\ \sum_{p=0}^{\frac{N-3}{2}} \left(\prod_{q=0}^p \left(1 - \frac{1}{S-2q} \right) \right) \frac{1}{S-2p-2}, & \text{when } N \text{ is odd} \\ 0, & \text{when } N = 2. \end{cases}$$

and

$$\kappa = \begin{cases} \sum_{p=1}^{\frac{N-2}{2}} \left(\prod_{q=1}^p \left(1 - \frac{1}{S-2q+1} \right) \right) \frac{1}{(S-2p)(S-2p+1)}, & \text{when } N \text{ is even} \\ \sum_{p=0}^{\frac{N-3}{2}} \left(\prod_{q=0}^p \left(1 - \frac{1}{S-2q} \right) \right) \frac{1}{(S-2p)(S-2p-1)}, & \text{when } N \text{ is odd} \\ 0, & \text{when } N = 2. \end{cases}$$

Under the standard assumptions that $S \geq N+1$ and $N \geq 2$, the buyer will prefer the reverse auction to the alternating auction if and only if $B > \frac{1+\varrho}{1-\varrho}$.

Proof: See Appendix B.

Table 1: Conditions for Forward and Reverse Auctions as Equilibrium Auction Preference

Player	Reverse	Forward
Supplier	$B > \frac{1+\gamma}{1-\gamma} : RA > FA$	$B < \frac{1+\gamma}{1-\gamma} : FA > RA$
Supplier	$B > \frac{1+\varrho}{1-\varrho} : RA > RFA$	$B > \frac{1+\eta}{1-\eta} : FA > RFA$
Buyer	$B > \frac{N}{\lambda} - 1 : RA > FA$	$B < \frac{N}{2\rho} - 1 : FA > RFA$
Buyer	$B > \frac{N}{2(\lambda-\rho)} - 1 : RA > RFA$	$B < \frac{N}{\lambda} - 1 : FA > RA$

Table 2: Conditions for Alternating Auction as Equilibrium Auction Preference

Player	Alternating
Supplier	$B < \frac{1+\varrho}{1-\varrho} : RFA > RA$
Supplier	$B < \frac{1+\eta}{1-\eta} : RFA > FA$
Buyer	$B > \frac{N}{2\rho} - 1 : RFA > FA$
Buyer	$B < \frac{N}{2(\lambda-\rho)} - 1 : RFA > RA$

Proposition 3.5.6 (*Supplier Forward Auction vs. Alternating Auction*) A profit-maximizing supplier will be indifferent between a forward auction and an alternating auction when

$$B = \frac{1+\eta}{1-\eta},$$

where

$$\eta = \frac{\frac{2(S-1)}{S+1} + 2\kappa S(S-1)}{(\alpha - \tau)S(S-1) - 1} + 1,$$

where α , κ and τ are as defined in Propositions 3.5.4 and 3.5.5.

Under the standard assumptions that $S \geq N+1$ and $N \geq 2$, the supplier will prefer the forward auction to the alternating auction if and only if $B > \frac{1+\eta}{1-\eta}$.

Proof: See Appendix B.

The conditions that must be satisfied to yield an equilibrium auction preference are in Tables 1 and 2. When the inequalities under the column heading 'Reverse' hold, the reverse auction (RA) is the equilibrium auction design. Likewise, when the inequalities under the headings 'Forward' and 'Alternating' hold the forward (FA) and alternating (RFA) auction designs are the equilibrium, respectively.

Table 3: Conditions when Alternating Auction is a Compromise

Supplier	Buyer
RA>RFA>FA	FA>RFA>RA
$B > \frac{1+\rho}{1-\rho}$	$B < \frac{N}{2\rho} - 1$
$B < \frac{1+\eta}{1-\eta}$	$B < \frac{N}{2(\lambda-\rho)}$
FA>RFA>RA	RA>RFA>FA
$B < \frac{1+\rho}{1-\rho}$	$B > \frac{N}{2\rho} - 1$
$B > \frac{1+\eta}{1-\eta}$	$B > \frac{N}{2(\lambda-\rho)}$

When neither the reverse nor the forward auction is an equilibrium, we consider the second choice of suppliers and buyers. If, for example, one side of the market prefers RA and the other prefers FA, it might be that RFA is an acceptable "compromise" as the second choice of each. In other words, we look to see whether $\text{Min}(RA, FA) < RFA < \text{Max}(RA, FA)$ for both sides of the market.

To determine if the space where these inequalities hold is nonempty, we consider the space of up to twenty suppliers and twenty periods. We replace the inequalities with equalities and the resulting equations give the number of buyers that would make each side indifferent between two given auction designs. When plotted in a three-dimensional space, the equation corresponds to an indifference surface with a unique value of B for each (N, S) pair. This surface illustrates a preference between two auction designs as the number of suppliers and the number of periods change. For values of B above the surface, one auction design is preferred while for values of B below the surface, the second of the two auction designs is preferred.

The indifference surfaces are displayed in Figures 14-16 and 19-21 of Appendix C. The graphs have asymptotes along the diagonal where the values of S and N approach one another. These asymptotes are a graphical default. The diagonal separates valid auctions (those with $S > N$) from invalid auctions (those with $S \leq N$). The surfaces oscillate as the number of periods change due to the stepwise nature of the functions. The corresponding values of B for the first twenty periods when there are less than 20 or fewer suppliers are displayed in the tables in Figures 36, 37, and ???. These tables aide in understanding the

behavior of the indifference surfaces when the number of suppliers is from 3 to 20 and the number of periods is from 2 to 20. The naming of the surface SRARFA, for example, refers to the surface for which the supplier is indifferent between the reverse auction(RA) and the alternating auction (RFA). Because we set the inequality to an equality, the value of SRARFA is the number of buyers participating in the exchange at which the supplier or buyer is indifferent between the two auction designs.

There are numerous intersections between buyer and supplier indifference surfaces for low values of S and N due to the relatively large fluctuations in the expected profit and surplus when N is small. The introduction of the sum and sum-product term into the expected profit and surplus functions as $N = 3$ and $N = 4$ affect these fluctuations in accordance with Propositions 3.4.2-3.4.6 and 3.4.8. The greatest fluctuations are on the side of the buyer and are outlined in the cases of Propositions 3.5.1-3.5.3. Prior to the introduction of the sum and sum product term, the alternating and conventional auction models are the same. As N continues to increase, the contribution of each additional period (as shown in Appendix D) is small relative to this initial introduction of the sum and sum-product terms. This allows both the supplier and buyer indifference surfaces to remain in their respective regions in the test range. The long-term behavior of the indifference surfaces is illustrated in Figure 1.

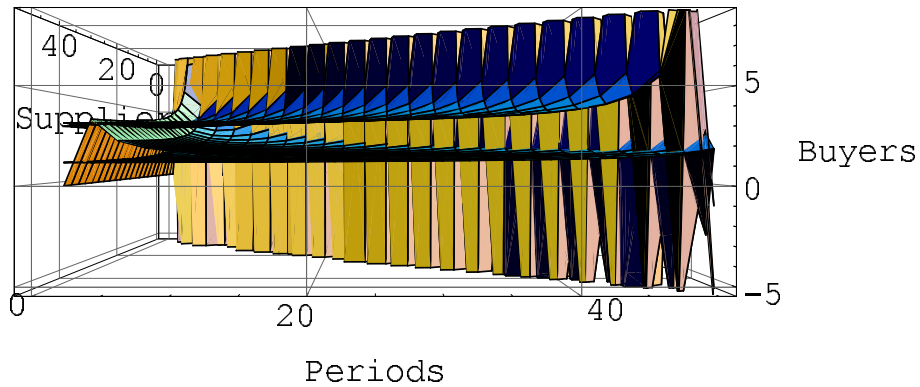


Figure 1: Indifference Surfaces for up to 50 Periods

Observation 1 For $N < 50$, from the perspective of the supplier, the indifference surfaces remain between 3 and 5 buyers. The indifference surfaces for the buyer remain

between 1 and 3 buyers. We observe that as N increases, the supplier and buyer indifference surfaces separate while the indifference surfaces that correspond to each market side become closer. The space between the surfaces corresponds to the region where neither the forward nor reverse auction serves as an equilibrium and where the alternating auction is a compromise. This space contains the integer value of three buyers ($B = 3$) for $N > 7$. Therefore, the space is non-empty for $N > 7$. The number of buyers B is an integer value within the region where RFA is a compromise. In this region neither RA nor FA was an equilibrium and RFA was a second choice for both sides of the market. This indicates that in expectation, the RA or FA model will be most likely an equilibrium outside of this region (i.e., $B < 3$ or $B > 3$).

A primary observation is that the presence of 3 buyers is a turning point for the supplier in equilibrium auction preference. This is because the expected price in the auction is the second highest bid, which is $\frac{B-1}{B+1}$. When $B = 3$, this term equals $\frac{1}{2}$. Because the bids can range from zero to one and the expected valuation is $\frac{1}{2}$, the forward auction is only profitable for the supplier when the number of buyers is greater than three. When suppliers know their valuation before they choose the auction design, if $B > \frac{1+v}{1-v}$, suppliers prefer to initiate a forward auction. On the other hand, when there are three or fewer buyers, suppliers and buyers prefer the RA auction type. This is because when the number of buyers is three or less, the suppliers are more susceptible to a negative profit in employing a FA. In this case the supplier prefers to bid in a RA, which guarantees the supplier a non-negative profit.

3.5.1.1 Additional Observations of Preferences

We illustrate the trend of the indifference surfaces when N is greater than seven periods within our twenty-period space. We chose to illustrate the trend for seven to twenty periods because the proofs from Propositions 3.5.1-3.5.6 give that the preferences of suppliers and buyers do not switch after $N > 7$. In addition, the tables and graphs in Appendix C show that the supplier indifference surfaces do not intersect the buyer indifference surfaces for N greater than seven within this space.

Observation 2 The indifference surfaces for the supplier cross as the number of periods

change from even to odd in the test range.

The tables in Figures 17 and 18 of Appendix C show how the indifference surface SFARFA crosses the other two surfaces, SRAFA and SRARFA, as the number of periods change from even to odd. The negative numbers that are above the diagonal correspond to regions of intersection. The tables give the difference between the values of B that make the supplier indifferent for each pair of auction designs. Therefore, the change in this difference from positive to negative or vice versa indicates an intersection between indifference surfaces. The negative values have been highlighted to indicate an intersection as either the value of S or N change. Because the change occurs at integer points the the preference at those points will be the dominant preference when all three surfaces are compared.

Observation 3 For the buyer, the BRARFA and BFARFA indifference surfaces also cross each other as the number of periods change from even to odd but neither crosses the BRAFA indifference surface in the test range.

The tables in Figures 22 and 23 of Appendix C demonstrate this as the number of periods change from even to odd.

When we consider the tables in Figures 31-35 in Appendix C we see that the buyer and supplier indifference surfaces all intersect within the first six periods and (except for the BFARFA surface) when the number of suppliers is less than twelve. The BFARFA surface is above the SRAFA and SRARFA surfaces when there are four periods even when the number of suppliers is above twelve. The result of the intersections is a change in equilibrium preferences. For example, if we compare the table values when there are two periods and five suppliers, the buyer indifference surfaces, RA/RFA and RA/FA, which tend to stay below the supplier indifference surfaces, are both above all supplier indifference surfaces and the buyer FA/RFA indifference surface. This affects the combined market equilibrium.

Observation 4 The indifference surfaces in the test range reach asymptotes along the diagonal of each surface as the values of S and N approach one another.

The differences in the expected return with an increase in the number of periods are in Appendix D. From the differences, we see that when the number of periods and the number of suppliers are close in value, the contribution of an additional period to the expected

return is greater.

Observation 5 The expected return from the alternating auction is non-negative in the test range for $B = 3$.

The tables in Figure 2 show the expected profit and surplus when $B = 3$ in the test range. This expected profit is non-negative for both sides of the market. The expected profit decreases for the supplier while the expected surplus increases for the buyer as the number of suppliers increases. This is because there is an increase in the competition between suppliers. We conclude the following

Proposition 3.5.7 (*Equilibrium Auction Design*) *The conditions for each auction design to be an equilibrium auction design for the N -period PDSM market are as listed in Table 1. When neither the reverse nor the forward auction design is an equilibrium, the alternating auction can serve as a profitable next best option and alternative to not conducting an auction at all.*

	Supplier Expected Profit from RFA Auction when B=3																			
	S=3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20		
N=2	0.04	0.01	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
3		0.06	0.04	0.02	0.02	0.01	0.01	0.01	0	0	0	0	0	0	0	0	0	0		
4			0.06	0.04	0.02	0.02	0.01	0.01	0.01	0	0	0	0	0	0	0	0	0		
5				0.07	0.05	0.03	0.03	0.02	0.01	0.01	0.01	0.01	0	0	0	0	0	0		
6					0.08	0.05	0.04	0.03	0.02	0.01	0.01	0.01	0.01	0	0	0	0	0		
7						0.08	0.06	0.04	0.03	0.02	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0		
8							0.08	0.06	0.04	0.03	0.02	0.02	0.02	0.01	0.01	0.01	0.01	0.01		
9								0.08	0.06	0.04	0.03	0.03	0.02	0.02	0.02	0.01	0.01	0.01		
10									0.08	0.06	0.04	0.03	0.03	0.02	0.02	0.01	0.01	0.01		
11										0.08	0.06	0.05	0.04	0.03	0.02	0.02	0.02	0.01		
12											0.08	0.06	0.05	0.04	0.03	0.02	0.02	0.02		
13												0.08	0.06	0.05	0.04	0.03	0.02	0.02		
14													0.08	0.06	0.05	0.04	0.03	0.02		
15														0.08	0.06	0.05	0.04	0.03		
16															0.08	0.06	0.05	0.04		
17																0.08	0.06	0.05		
18																	0.08	0.06		
19																		0.08		
20																				

							Buyer Expected Surplus from RFA Auction when B=3																
	S=3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20					
N=2	0.08	0.11	0.13	0.15	0.16	0.17	0.18	0.18	0.19	0.19	0.2	0.2	0.2	0.21	0.21	0.21	0.21	0.21					
3		0.12	0.15	0.18	0.19	0.2	0.21	0.22	0.23	0.23	0.24	0.24	0.24	0.25	0.25	0.25	0.25	0.25					
4			0.22	0.27	0.3	0.33	0.35	0.36	0.38	0.39	0.4	0.41	0.41	0.41	0.42	0.42	0.42	0.43					
5				0.26	0.31	0.34	0.37	0.39	0.4	0.41	0.42	0.43	0.44	0.45	0.45	0.46	0.46	0.47					
6					0.38	0.44	0.48	0.52	0.54	0.56	0.58	0.59	0.6	0.61	0.62	0.63	0.63	0.64					
7						0.43	0.48	0.53	0.56	0.58	0.6	0.62	0.63	0.64	0.65	0.66	0.67	0.67					
8							0.57	0.63	0.68	0.71	0.74	0.76	0.78	0.8	0.81	0.82	0.84	0.84					
9								0.61	0.67	0.72	0.76	0.78	0.81	0.83	0.84	0.85	0.87	0.88					
10									0.76	0.83	0.88	0.92	0.95	0.98	1	1.01	1.03	1.04					
11										0.8	0.87	0.92	0.96	0.99	1.02	1.04	1.06	1.07					
12											0.96	1.04	1.09	1.13	1.16	1.19	1.21	1.23					
13												1.01	1.08	1.13	1.17	1.2	1.23	1.25					
14													1.17	1.25	1.3	1.35	1.38	1.41					
15														1.21	1.29	1.34	1.39	1.42					
16															1.39	1.46	1.52	1.56					
17																	1.43	1.5	1.56				
18																		1.6	1.68				
19																				1.64			
20																						1.71	

Figure 2: Indifference Surfaces for up to 50 Periods

3.5.2 Comparative Statics

We are also interested in the effect of a local increase in the number of periods on the expected profit and surplus. For example, how valuable is it to an auctioneer to extend the number of periods by one? The answer to this question will help the auctioneer determine how many periods to include in the auction considering any fees, advertising, or opportunity costs involved in holding an auction. For example, an auctioneer could extend the number of periods in the sequence until the increase in the expected profit or surplus from an additional period becomes less than the variable or opportunity costs of conducting an auction in that period.

The amount of increase in expected profit or surplus can be determined by taking the difference in the expected return (i.e., $P_{N+1} - P_N$). This gives the contribution of the addition period to the expected profit or surplus. This analysis for the reverse, forward and alternating cases is in Appendix D.

We found that the expected profit to a supplier from a reverse auction increases with an increase in the number of periods. The expected profit to a supplier from a forward auction decreases with an increase in the number of periods when $B = 2$. This is because when there are only two buyers, the expected winning price of each auction is less than the supplier's expected private valuation. The expected profit to a supplier from a forward auction is non-decreasing when $B \geq 3$. The expected profit to a supplier from an additional period in an alternating auction design increases when $B \geq 3$ and is non-increasing when $B = 2$.

For the buyer, the expected surplus from a reverse auction increases with an increase in the number of periods as long as the difference between the number of suppliers and period is greater than three. After this point, it is non-increasing. The expected surplus to the buyer from a forward auction increases with an increase in the number of periods. The expected surplus to the buyer from an increase in the number of periods in an alternating auction increases when the inequality $\frac{B+2}{2(B+1)} > \frac{2S-2N+4+B}{(S-N+1)(S-N+2)}$ holds, when otherwise it is non-increasing. Specifically, when $B + 2 > (S - N + 1)(S - N)$, an increase in N decreases the expected surplus.

Table 4: Model Comparison - Change in Expected Return With an Increase in the Number of Periods

Models	Supplier	Buyer
RA	\uparrow	\uparrow *
FA	$\leftrightarrow, \uparrow^{**}$	\uparrow
RFA	\uparrow^{***}	\downarrow^{****}

*when $S - N > 3$; otherwise it's non-increasing

**when $B \geq 3$; otherwise it's decreasing

***when $B \geq 3$; otherwise it's non-increasing

****when $\frac{B+2}{2(B+1)} > \frac{2S-2N+4+B}{(S-N+1)(S-N+2)}$; otherwise it's decreasing

Therefore, when $(S - N) \leq 3$, there exists an upper bound on the number of periods that both buyers and supplier prefer the RA model. When $B = 2$, there is an upper bound on the number of periods that both buyers and supplier prefer for the FA model. For the RFA model, an upper bound exists when $B = 2$ and $S - N \geq 3$ and when $B \geq 3$ and $S - N = 1$. In the cases that an upper bound does not exist, as S and N approach one another the bound is formed. The values of S , B , and N determine when an upper bound exists. A summary of the rate of an increase in the number of periods on the expected profit and surplus is in Table 4.

We conclude the following

Proposition 3.5.8 (*Preference of Sequence Length*) *The effect of increasing the number of auction periods is shown in Table 4.*

Proof: See Appendix D.

3.5.3 Expected Value of Information

In this chapter, we determine the expected value of learning the future valuation for the supplier. One of the primary assumptions of the PDSM model is that the private valuations change each period. Given the uncertainty of private valuations in the future, we are interested in how much a supplier would be willing to pay to learn her future private valuation. This is important for the supplier because her bid strategy includes her expected future profits. Eliminating the uncertainty in the future, can help the supplier behave

strategically across periods. We consider the expected profit to the strategic supplier after she has chosen to participate in a two-period auction type for two cases. We calculate the expected value of the option to learn the private valuation in the last period both before and after learning the valuation in the 2^{nd} -to-last period for each auction model.

As we showed in Chapter 3.4, for the PDSM FA and RFA models, the bid strategy for the buyers is to bid truthfully in every period. Therefore, knowledge about future valuations will not affect the expected surplus to the buyer. However, the bid function for the suppliers in the PDSM RA and RFA models equals the private valuation inflated by a function of S , B and N .

Let v_2 and v_1 be the valuation in the 2^{nd} -to-last and last period, respectively. To determine the expected value to a supplier of knowing her second period valuation at the beginning of the 2^{nd} -to-last period auction given that she has already chosen to participate in a two-period auction of a specific type, we determine the difference of the expected profit when v_1 is known and unknown for the reverse and alternating model because it is in these models that the supplier can behave strategically.

We then determine the expected profit from each outcome of perfect information by integrating over the range of values of v_1 and v_2 that produce each outcome when both v_1 and v_2 are known. When v_1 is not known, we fix v_1 and determine the expected profit from each outcome of perfect information. To determine the expected value to a supplier of learning what her private valuation will be in the last period *after* she has discovered what her valuation is in the 2^{nd} -to-last period, we integrate over the values of v_1 . To determine the expected value to the supplier of learning what her private valuation will be in the last period *before* discovering her valuation in the 2^{nd} -to-last period, we integrate over both v_1 and v_2 respective to each outcome of perfect information.

To determine the expected value to a supplier of learning the value of v_1 both before and after learning the value of v_2 , we sum the conditional expected profit from each outcome of perfect information and subtract the expected profit when v_1 is unknown.

Proposition 3.5.9 (*Expected Value of Information*) *For the reverse auction, the expected value to the supplier of knowing her valuation in the last period after choosing to participate*

in a two-period auction after (EVI_A) and before (EVI_B) learning her valuation in the 2nd-to-last period are as follows:

When $v_2 > \frac{S-1}{S}$,

$$\begin{aligned} EVI_A(v_2) &= \left(\frac{(1-v_2)^S}{S(S-1)} \right) + \left(\frac{(1-Z)^S}{S(S-1)} \right) \\ &+ \left(\frac{1}{S} \left[\sum_{i=0}^S (1-v_2 + \frac{1}{S(S-1)})^i \binom{S}{i} \left[\frac{-(1-Z)^{(S-1)(S-i)+1}}{[(S-1)(S-i)+1](S-1)^{S-i}} \right] \right] \right) \\ &- \left(\frac{(1-v_2)^S}{S} + \frac{1}{S(S-1)} \right). \end{aligned}$$

When $v_2 < \frac{1}{S(S-1)}$,

$$\begin{aligned} EVI_A(v_2) &= \left(\frac{1}{S-1} - v_2 - \frac{W}{S-1} + v_2 W \right) + \frac{1-(1-W)^S}{S(S-1)} \\ &+ \left(\frac{1}{S} \left[\sum_{i=0}^S (1-v_2 + \frac{1}{S(S-1)})^i \binom{S}{i} \left[\frac{(1-W)^{(S-1)(S-i)+1} - 1}{[(S-1)(S-i)+1](S-1)^{S-i}} \right] \right] \right) \\ &- \left(\frac{(1-v_2)^S}{S} + \frac{1}{S(S-1)} \right). \end{aligned}$$

When $\frac{1}{S(S-1)} \leq v_2 \leq \frac{S-1}{S}$,

$$\begin{aligned} EVI_A(v_2) &= \left(\frac{1}{S} \left[\sum_{i=0}^S (1-v_2 + \frac{1}{S(S-1)})^i \binom{S}{i} \left[\frac{-1}{[(S-1)(S-i)+1](S-1)^{S-i}} \right] \right] \right) \\ &- \left(\frac{(1-v_2)^S}{S} \right). \end{aligned}$$

and

$$\begin{aligned} EVI_B &= \left[\left(\frac{-(1-v_2)^{S+1}}{S(S-1)(S+1)} \right) \Big|_{\frac{S-1}{S}}^1 \right] + \left(\frac{[\frac{1}{S} + (1-v_2)(S-1)]^{\frac{S}{S-1}+1}}{S(S-1)(\frac{S}{S-1}+1)} \frac{-1}{S-1} \Big|_{\frac{S-1}{S}}^1 \right) \\ &+ \int_{\frac{S-1}{S}}^1 \left(\frac{1}{S} \left[\sum_{i=0}^S (1-v_2 + \frac{1}{S(S-1)})^i \binom{S}{i} \left[\frac{-[\frac{1}{S} + (1-v_2)(S-1)]^{(S-i)+\frac{1}{S-1}}}{[(S-1)(S-i)+1](S-1)^{S-i}} \right] \right] \right) dv_2 \\ &- \left(\frac{(1-v_2)^{S+1}}{S(S+1)} + \frac{v_2}{S(S-1)} \Big|_{\frac{S-1}{S}}^1 \right) \\ &+ \int_{\frac{1}{S(S-1)}}^{\frac{S-1}{S}} \left[\left(\frac{1}{S-1} - v_2 - \frac{1 - [\frac{1}{S} - v_2(S-1)]^{\frac{1}{S-1}}}{S-1} + v_2 [1 - [\frac{1}{S} - v_2(S-1)]^{\frac{1}{S-1}}] \right) \right. \\ &+ \left(\frac{1 - [\frac{1}{S} - v_2(S-1)]^{\frac{S}{S-1}}}{S(S-1)} \right) \\ &+ \left(\frac{1}{S} \left[\sum_{i=0}^S (1-v_2 + \frac{1}{S(S-1)})^i \binom{S}{i} \left[\frac{[\frac{1}{S} - v_2(S-1)]^{(S-i)+\frac{1}{S-1}} - 1}{[(S-1)(S-i)+1](S-1)^{S-i}} \right] \right] \right) \\ &- \left(\frac{(1-v_2)^S}{S} + \frac{1}{S(S-1)} \right) \Big] f(v_2) dv_2 \\ &+ \int_0^{\frac{1}{S(S-1)}} \left[\left(\frac{1}{S} \left[\sum_{i=0}^S (1-v_2 + \frac{1}{S(S-1)})^i \binom{S}{i} \left[\frac{-1}{[(S-1)(S-i)+1](S-1)^{S-i}} \right] \right] \right) \right. \\ &- \left(\frac{(1-v_2)^S}{S} \right) \Big] f(v_2) dv_2. \end{aligned}$$

For the alternating auction, the expected value to the supplier of knowing her valuation in the last period after choosing to participate in a two-period auction after (EVI_A) and before (EVI_B) learning her valuation in the 2nd-to-last period are as follows:

When $v_2 > 1 - \frac{1}{2(S-1)}$,

$$\begin{aligned}
EVI_A(v_2) = & \left(\frac{\frac{B-1}{B+1}X - \frac{X^2}{2}}{S-1} \right) \\
& + \left(\frac{\frac{B-1}{B+1} - \frac{1}{2}}{S-1} + \frac{(1-v_2 - \frac{\frac{B-1}{B+1}-1}{S-1} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \right. \\
& \left. - \left(\frac{\frac{B-1}{B+1}X - \frac{X^2}{2}}{S-1} + \frac{(1-v_2 - \frac{\frac{B-1}{B+1}-X}{S-1} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \right) \right) \\
& - \left[(1-v_2)^{S-1} \left(\frac{B-3}{2(B+1)(S-1)} \right) + \frac{(1-v_2)^S}{S} + \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S} \frac{(1-(1-v_2)^{S-1})}{S-1} \right].
\end{aligned}$$

When $v_2 < \frac{1}{2(S-1)}$,

$$\begin{aligned}
EVI_A(v_2) = & \left(\frac{B-1}{2(B+1)(S-1)} - v_2 + \frac{1}{S} \right) - \left(\frac{B-1}{2(B+1)(S-1)} Y - v_2 Y + \frac{Y}{S} \right) \\
& + \left(\frac{\frac{B-1}{B+1}Y - \frac{Y^2}{2}}{S-1} + \frac{(1-v_2 - \frac{\frac{B-1}{B+1}-Y}{S-1} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \right) \\
& - \left(\frac{(1-v_2 - \frac{B-1}{(B+1)(S-1)} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \right) \\
& - \left[(1-v_2)^{S-1} \left(\frac{B-3}{2(B+1)(S-1)} \right) + \frac{(1-v_2)^S}{S} + \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S} \frac{(1-(1-v_2)^{S-1})}{S-1} \right].
\end{aligned}$$

When $\frac{1}{2(S-1)} \leq v_2 \leq 1 - \frac{1}{2(S-1)}$,

$$\begin{aligned}
EVI_A(v_2) = & \left(\frac{\frac{B-1}{B+1} - \frac{1}{2}}{S-1} + \frac{(1-v_2 - \frac{\frac{B-1}{B+1}-1}{S-1} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \right) \\
& - \left(\frac{(1-v_2 - \frac{B-1}{(B+1)(S-1)} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \right) \\
& - \left[(1-v_2)^{S-1} \left(\frac{B-3}{2(B+1)(S-1)} \right) + \frac{(1-v_2)^S}{S} + \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S} \frac{(1-(1-v_2)^{S-1})}{S-1} \right].
\end{aligned}$$

and

$$\begin{aligned}
EVI_B = & \int_{1-\frac{1}{2(S-1)}}^1 \left[\left(\frac{\frac{B-1}{B+1}X - \frac{X^2}{2}}{S-1} \right) \right. \\
& + \left(\frac{\frac{B-1}{B+1} - \frac{1}{2}}{S-1} + \frac{(1-v_2 - \frac{\frac{B-1}{B+1}-1}{S-1} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \right. \\
& \left. \left. - \left(\frac{\frac{B-1}{B+1}X - \frac{X^2}{2}}{S-1} + \frac{(1-v_2 - \frac{\frac{B-1}{B+1}-X}{S-1} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \right) \right) \right] f(v_2) dv_2 \\
& - \left[(1-v_2)^{S-1} \left(\frac{B-3}{2(B+1)(S-1)} \right) + \frac{(1-v_2)^S}{S} + \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S} \frac{(1-(1-v_2)^{S-1})}{S-1} \right] f(v_2) dv_2 \\
& + \int_0^{1-\frac{1}{2(S-1)}} \left[\left(\frac{B-1}{2(B+1)(S-1)} - v_2 + \frac{1}{S} \right) - \left(\frac{B-1}{2(B+1)(S-1)} Y - v_2 Y + \frac{Y}{S} \right) \right. \\
& + \left(\frac{\frac{B-1}{B+1}Y - \frac{Y^2}{2}}{S-1} + \frac{(1-v_2 - \frac{\frac{B-1}{B+1}-Y}{S-1} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \right) \\
& \left. - \left(\frac{(1-v_2 - \frac{B-1}{(B+1)(S-1)} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \right) \right] f(v_2) dv_2 \\
& - \left[(1-v_2)^{S-1} \left(\frac{B-3}{2(B+1)(S-1)} \right) + \frac{(1-v_2)^S}{S} + \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S} \frac{(1-(1-v_2)^{S-1})}{S-1} \right] f(v_2) dv_2 \\
& + \int_{1-\frac{1}{2(S-1)}}^1 \left[\left(\frac{\frac{B-1}{B+1} - \frac{1}{2}}{S-1} + \frac{(1-v_2 - \frac{\frac{B-1}{B+1}-1}{S-1} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \right) \right. \\
& \left. - \left(\frac{(1-v_2 - \frac{B-1}{(B+1)(S-1)} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \right) \right] f(v_2) dv_2 \\
& - \left[(1-v_2)^{S-1} \left(\frac{B-3}{2(B+1)(S-1)} \right) + \frac{(1-v_2)^S}{S} + \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S} \frac{(1-(1-v_2)^{S-1})}{S-1} \right] f(v_2) dv_2.
\end{aligned}$$

Proof: See Appendix E.

Given the expected value of information equations, a supplier can determine if it is worth knowing her future valuation for different values of S and B . Because it is possible for the supplier who initiates a forward auction to receive a negative profit, she would prefer to bid accordingly in the first auction of the alternating sequence to maximize her expected profit.

When the RA model is used, the bid of the supplier in the 2^{nd} -to-last period depends on her private valuation in the last period. If the supplier loses the 2^{nd} -to-last period auction, she will bid truthfully in the last period. In the combined reverse and forward auctions, the supplier will bid according to the expected profit in the last period. In this respect, the RFA model and RA model are similar. In the RFA model the expected winning bid price in the last period is the same regardless of the supplier who initiates but the expected profit depends on a supplier's private valuations if she uses the option to learn it beforehand. If

she learns her valuation before hand and finds that there will be a negative expected profit in the last period, this will affect the bid placed in the 2^{nd} -to-last period. The bid in the 2^{nd} -to-last period will likely be more aggressive to avoid a future loss.

If suppliers initiate a forward auction, before knowing their future private valuation, they may have a positive expected profit going into the sequential auction and later regret their decision since they cannot enforce a reservation price above their cost. This could be a costly mistake for firms who are new to auctions to suffer because they may later find that they cannot enforce their reservation price.

3.6 Conclusions

In this chapter, we characterized the expected return and bid strategy of a market participant in three models of sequential auctions. We developed a new hybrid auction model (RFA) and compared its expected return to that of two conventional models (RA and FA). The hybrid and conventional models of expected return are similar when there are only a few (one or two) periods in the sequence.

We generalized the auction models to provide an N -period formulation of the optimal bid strategy. The Bayesian-Nash equilibrium is for each bidder to bid according to an optimal bid strategy. In our models, the bid strategy for the buyer is to bid his private valuation. The bid strategy for the supplier is to inflate her bid above her private valuation. The amount of the inflation depends on the sequential auction design and auction parameters.

We defined the conditions under which each auction design is an equilibrium. We also defined the conditions under which neither the reverse nor the forward auction design is an equilibrium. When neither is an equilibrium, we found where the alternating auction is a second choice within a sample region. In the sample region, the alternating auction produced positive return and was therefore better than holding no auction at all.

We further investigated these equilibrium conditions using a subset of values for the number of suppliers and auction periods in the sequence. In doing so, we observed that the indifference surfaces for buyers and suppliers intersect when the number of periods is small (seven periods or less) but a distinct difference between buyer and supplier indifference

surfaces becomes apparent as the number of periods increase. We also observed that both the indifference surfaces of a supplier intersect as the number of periods change from even to odd and vice versa. Only two of the three buyer indifference surfaces intersect as the periods change. As the number of periods increase, the indifference surfaces for suppliers and buyers are above and below a value of three for B , respectively. In addition, the space where the alternating auction is a compromise is non-empty.

We used the difference in the expected return between periods to determine the effect of increasing the number of periods on the expected supplier profit and buyer surplus. We found the conditions under which the expected return to a supplier and buyer change with an increase in the number of periods. The presence of three buyers tends to affect the expected return in auction designs in addition to the number of periods. We also found the bounds on the length of the auction sequence under each auction design.

We also determined the expected value of knowing the private valuation in the last period of a two-period sequence after a commitment to an auction design has been made. Because suppliers are subject to a negative expected profit when participating in a forward PDSM auction, it is likely in their best interest to learn their future private valuation. It is important that they know how much they should pay for that information.

Some extensions of this work involve relaxing the assumption that the sequences alternate and searching for properties of an equilibrium participation strategy when any supplier or buyer desires to initiate in any period. When a supplier initiates an auction, her private valuation influences how profitable the auction will be. In the models, we assume that the future valuations are random. However, if the supplier learns her future valuations at the beginning of the sequence, she may not want to initiate a forward auction. In this case, the option to initiate an auction or bid will follow a mixed strategy in equilibrium. It would also be useful to include alternating sequences in the larger picture of a total sourcing policy.

CHAPTER IV

KEY FACTORS IN SUCCESSFUL TIMBER PROCUREMENT AUCTIONS

We analyzed data from reverse auctions for timber procurement on the spot market and found less than half of the auctions that were initiated attracted one or more bidders. Because aggregating volume is a primary goal of auctioneers for using the auction, we look for plausible explanations using factors that tend to affect supply. We look at the effect of factors that are internal and external to the auction to determine their influence on the supply of timber transacted in the online spot market. We found that the number of bidders had the largest independent influence on the amount of supply transacted on the exchange. We also found a seasonal effect on the amount supplied but no incentive for suppliers to withhold supply for price benefits.

4.1 Literature Review

Timber has been the highest valued crop in 8 Southern states and it has ranked among the top three agriculture crops in all of the 13 Southern states (USDA, 1988). The US is the largest producer of industrial roundwood in the world, almost double any other country. This is true despite the fact that it has only 6% of the world's forest area. The South's part in this is substantial. It produces about 55% of the total US harvest and has about one-half the total industrial forest plantations in the world (Cubbage et al., 1998). The supply chain for the pulp and paper industry differs in the South and Pacific-Northwest, which are the main U.S. forest locations.

About 70% of all paper is produced in integrated pulp and paper mills, which turn timber into pulp and then into paper. Recycled fiber is gaining popularity as a substitute input for timber at pulp and paper mills. Due to government mandates and environmental regulations in the early 1990's, much of the recently built pulping capacity uses waste paper

instead of virgin wood fiber (Hillstrom, 1994). The share of recycled paper used by paper mills in the US is projected to increase from 25% in 1998 to 36% by the year 2040 while timber, also known as roundwood, consumption decreases from 50% to 30% (Skog et al., 1999). The balance includes fiber from mill residue and imported woodpulp. The amount of imported woodpulp is comparatively small at roughly 6% but may increase to as much as 21% by 2040. The process for using each type of input differs and most mills were designed with timber as the input.

Even though recycled fiber is environmentally friendly because it reduces landfill amounts, it is also harmful in that up to 50% of incoming recycled paper can be useless and must be discarded as sludge. In addition, the processing of recycled paper does not produce the black liquor or other by-products of wood-fiber processing that mills use to generate their own electricity (Hillstrom, 1994).

Integrated pulp and paper mills require timber to run, and must run at operating rates as high as 88% (Korutz, 2003). The mills store an inventory of cut timber that can last up to 3 months. The companies that own these mills also own timberland from which they cut timber as needed. However, if they are able to purchase cut and delivered timber online at a low price or they foresee a need for additional timber in the forecast that will exceed the inventory or contracted volumes for that period, they forego cutting from their timberlands and turn to spot markets to purchase additional timber. Although mills use domestic timber, imported woodpulp, recycled fiber and mill residue as fiber inputs for paper products, timber is currently the most cost-effective wood-fiber for filling spot demand. The price paid for the timber on the spot market includes the cutting and delivery of the logs and is referred to as the delivered price.

As popularity of recycled fiber input increases, the mills find it less profitable to own their own timberlands; this is especially true now that they have altered their production process to receive recycled inputs. Along with the costs and risks associated with holding timberlands, the projected fiber ratios may be an impetus for the shift in timberland ownership from industrial ownership to private ownership. The timber supplied from private sources is expected to grow by 67% compared to a growth of only 17% from other types of

ownership. In addition, imports of pulpwood and woodpulp are projected to remain small relative to domestic supply (Skog et al., 1999). As firms decrease their timberland holdings, they become more dependent on suppliers of domestic timber when the need for virgin fiber arises. In recent years, spot market transactions have moved online and suppliers place bids to sell timber in reverse auctions. Aside from turning to the spot market for ease and profitability of transactions, shifts in land ownership in the procurement supply chain increase the need for an understanding of what makes timber auctions successful. As auctioneers, mill management is interested in knowing what influences the supply that is transacted online.

In Chapters 4.1.1-4.1.3, we provide a general overview of the procurement process for timber in the southeastern United States, explain the adoption of online auctions for timber procurement in this market, and explain the auction environment from which our data was observed. In Chapter 4.2, we investigate the internal and external factors that tend to affect the volume of timber supplied through an online exchange. In Chapter 4.3, we outline the statistical analysis used to determine the influence of each factor on the amount of supply transacted online. In Chapter 4.4, we summarize the results and give industrial implications of the analysis. In Chapter 4.5, we provide suggestions for future data collection. In Chapter 4.6, we provide insight on possible extensions of this work.

4.1.1 Procurement Process

The procurement process is only a portion of the overall pulp and paper supply chain, which is depicted in Figure 3. The wood fiber input is converted into paper products at a mill then shipped to distributors and finally to retailers.

Traditionally, the primary source of timber for mills logging companies in the northern and western region of the United States has been the national forests. The timber sold from these forests by the U.S. Forest Service is sold in standardized open and sealed bid auctions. Because the information from the sale of these forests is public knowledge, there has been research on the bidding behavior with results on the role of private information in common value auctions where the volume harvested is unknown beforehand (Athey and

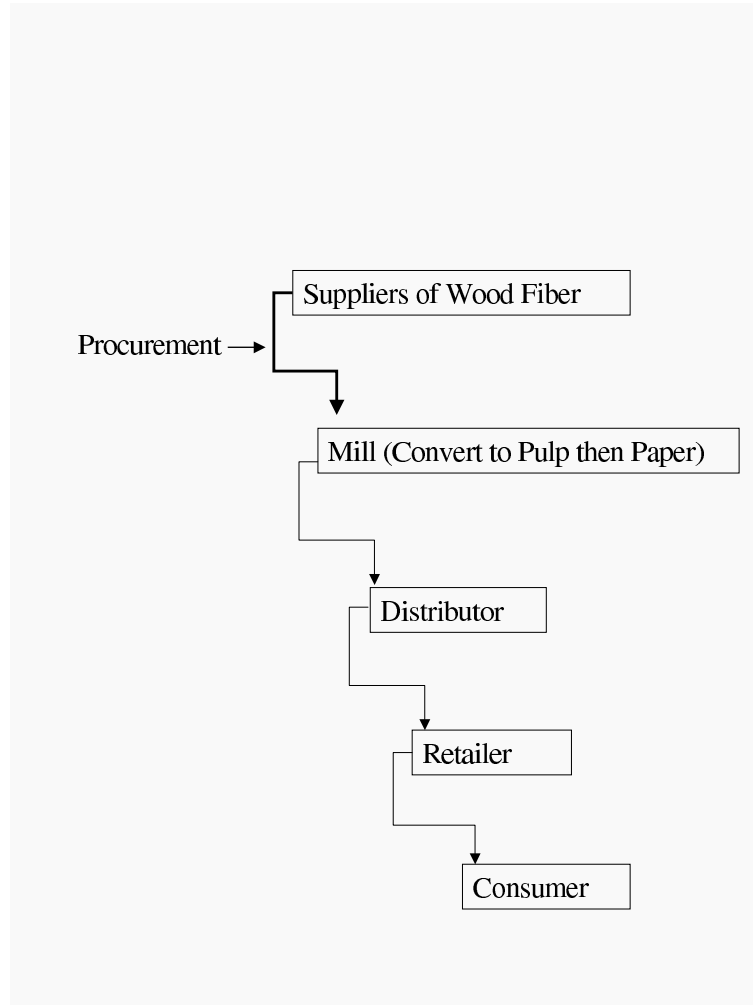


Figure 3: Pulp & Paper Supply Chain

Levin, 2001). There has also been work done on the relative performance of open and sealed bid auctions to find that sealed bid auctions attract more small bidders (Athey et al., 2004). This work corresponds to the activity on the right side of Figure 4. Our work is the first work that we are aware of that focuses on the bidding activity between the logging company and the mill.

The procurement process that we consider accounts for the transactions between the logging companies that resell timber purchased from owners of timberlands and the mill that converts the timber to paper products. In the United States, private individuals and firms together own roughly 71% of all timberland. The remaining 29% is owned by government or public agencies. Most timber purchased in the U.S. is purchased from thousands of private

landowners. Timberland owned by non-industrialized private landowners accounts for 47% of growing timber in the United States (USDA 1997).

Many landowners work with consultants to sell the rights to cut timber on their land. This sale is usually between the landowner and a timber company or a large pulp and paper company via a sealed bid auction. Timber companies add value to the timber, which is also known as pulpwood, by logging it and delivering it to mills of the pulp and paper companies (Figure 4). There are roughly 70,000 loggers in the United States and 37% of those work in the Southeast. These roughly 26,000 loggers work in crews of ten or less. Some of these roughly 2,600 crews work independently and some are aggregated by timber companies into groups of twenty or so to meet the demands of paper mills (USDL 2003).

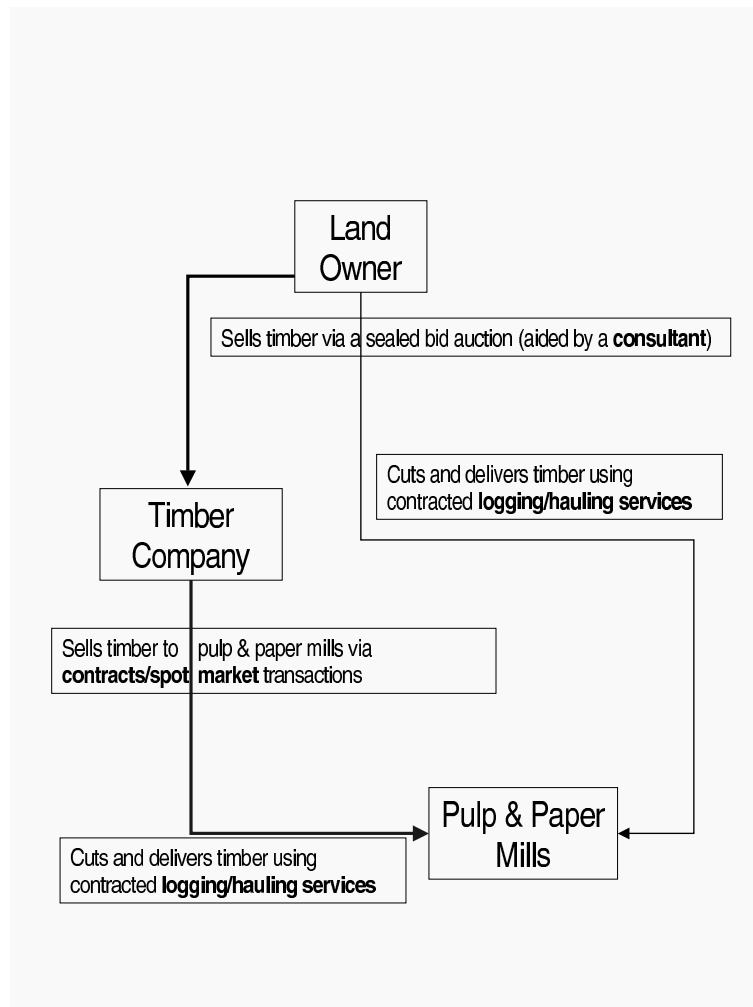


Figure 4: Procurement Process

The primary job of the timber company is to aggregate timber from numerous landowners to sell to the large pulp and paper companies. A timber company may contract with several logging contractors which allows them to purchase different sized tracts of timber to be logged and sold to a mill. Timber that is sold to a mill is typically logged within a 100 mile radius of that mill to keep transportation costs down. In 2002, 92 mills were operating and procuring wood from the 13 Southern states (Johnson and Steppleton 2004). There are numerous timber companies with differing sizes, alliances, business structures, which leaves them out of comprehensive vertical integration plans of large paper companies. Therefore, the management of these pulp and paper companies is tasked with gaining price and volume visibility using options offered by electronic commerce.

4.1.2 Adoption of Online Auctions

The relationships used in securing long-term contracts have also been used to secure spot market deals. When parties establish long-term relationships, they are more likely to take the time to negotiate agreeable contracts that define several terms of the relationship such as flexible price concessions, volume commitments, and product quality. It is reasonable for these long-term contracts to be relationship-driven for purposes of information sharing, customized pricing, and price adhesion where price changes trail supply or demand shocks (Grey et.al 2003). However, for short term contracts where it becomes more important to secure a price and/or quantity, an auction may prove to be the more beneficial trade mechanism. One critical factor in the successful implementation of an e-commerce tool is the auctioneer's sourcing strategy (Jap, 2002). The role of the Internet has increased in this regard in the past several years.

As Internet technology advances, companies in the pulp and paper industry have slowly adopted it for their use. In 1999, a survey revealed that the mill managers were beginning to access the Internet at the mill. The primary use of the Internet was to acquire industry news, product information, and communication with customers. Less than 20% of mill managers made purchases of any kind (Shaw, 1999). The trend of using the Internet to access information began to grow across the industry. Two years later, a survey of managers of

forest products companies in the United States identified benefits their companies received from conducting business electronically. The greatest benefits were access to industry information, real-time transactions, and exposure and access to potential customers (Vlosky, 2001).

A benefit of auctions is that they enable increased market visibility for participants. With adoption of online capabilities in the form of reverse auctions, companies hoped to achieve increased market visibility. When a buyer posts a request for some volume of demand, he is revealing information about the demand he faces in the supply chain. The hope is that the suppliers will aid the buyer in satisfying the demand. The buyer does not know whether the supply he sees in the form of price/quantity bids represents all of the available supply. Mills can benefit most from using a market mechanism that encourages suppliers to reveal their available supply since the mill's primary goal in using the reverse auction is the aggregation of timber supply to satisfy their demand.

The price that mills pay for timber from timber companies includes the price of the timber, the logging and the hauling costs. This price is referred to as the delivered price. The delivered price of the timber is influenced by the location of the forest from which it was logged. This location changes constantly as new tracts of timber are logged.

4.1.3 Overview of B2B Auction Environment

The timber procurement market structure has a high ratio of sellers to buyers. Auction intermediaries host web sites that allow buyers to initiate reverse auctions. One such site is ForestExpress.com. Through their online exchange for timber, the Trading Center, mills and timber suppliers can transact spot quantities of timber by way of reverse auctions. The format of the Trading Center allows auctioneers to design some properties of the auction as they choose including the duration, the list of invited bidders (which could include all of the site's registered suppliers), the quantity and description of the good, the reservation price, the bid increment, and the pricing rule. Auctioneers are able to choose among two pricing rules: discriminatory, where winners pay the amount of their bid and uniform, where winners pay the lowest accepted price.

We analyzed data on 507 of the reverse auctions conducted on the Trading Center for requested tons of delivered timber held over 10 consecutive months. Membership was required for participation in the online auction exchange. The bidders in each auction were timber companies who had purchased rights to cut timber on various tracts of land, which were initially owned by private landowners. The request for timber was posted online and the mill could invite as few as one bidder to bid. Only invited members that registered and met quality and delivery requirements could view or submit bids, but if no bidders were invited, the auction request was open to all members of the exchange. Of the 507 auctions that were initiated, 58% did not attract any bidders. Because other auctions had multiple winners, there were 320 separate transactions. The auctions were held at random times and were initiated by pulp and paper mills. These spot market auctions were for relatively small volumes.

Each auction typically lasted one week with expected delivery the following week. Each bid included the volume and price of the timber the timber company could supply. Although each request included an auction length, delivery location, bid increment, description of the good, volume, and reservation price, the archived data did not include information on the delivery location, the invited bidders nor the bid increment.

4.2 Characteristics of Successful Auctions

The volume of timber supplied via online auctions is influenced by factors that are either internal or external to the auction environment. Internal factors involve the outcome, rules, and design of the auction. They include activities in which a buyer or supplier may behave strategically such as in the setting of a reservation price or the decision to place a bid. The internal factors of the auction that might influence supply are the volume demanded, the reservation price, the description of the good, the type of good and the number of bidders invited. We consider the observed responses of the number that actually place bids and the number of winning bidders.

External factors affect the availability of the timber supply to the logging company. The external factors that might make participation in a spot market auction difficult or

unattractive include a reduced logging workforce as many loggers begin to retire, dangerous logging conditions, competing outlets for sales including satellite chip mills, the option to hold the timber rights on the spot amount to combine for a long-term contract, and little or no access to Internet technology.

Conversations with industry experts revealed one external factor that is presumed to have a large impact on the availability of timber. This factor being logging conditions, which are directly effected by weather patterns. We include additional data in our model to account for the impact of logging conditions on the timber supplied in the online reverse auction.

Our results indicate the extent to which the auction environment and logging conditions influence the amount of timber supplied through online reverse auctions. We now consider these internal and external factors in greater detail.

4.2.1 Seasonal Patterns

Seasonal patterns have been found to affect both the price and supply of timber. For example, stumpage prices of timber sold from national forests in the pacific northwest tend to be higher in the winter than in the summer. The price of cut timber in the northwest can be as much as 18% higher in the winter than in the summer (Haynes, 1998). Likewise, in the southeast, comparatively warmer winter temperatures combined with precipitation make the ground slippery and logging difficult. Conversely, in the northeastern United States, competition is likely to increase during the winter months because it is easier for loggers to remove trees when the ground freezes (Cooner, 2004). Therefore, the impact of weather is not uniform in the US. The south produces about 55% of the total US harvest (Cubbage et al., 1998), and the majority of our data came from auctions where the participants were in the south.

We use three measures to represent the effect of seasonal patterns on timber supply. They are the expected and actual monthly level of precipitation and the expected and actual Keetch-Byrum Drought Index (KBDI). The expected precipitation and KBDI are tested because loggers may make adjustments in external factors such as their labor force

and long-term contracts based on historical weather information. These adjustments may have a significant affect on the amount they are able to supply in the spot market.

We collected data on the expected and actual monthly precipitation averages from 1971-2000 for the southeastern states from the web site of the Southeast Regional Climate Center (SERCC, 2005). We categorized the months in which auctions were held as wet or dry according to where the corresponding precipitation values fell in relation to the median value. The method of evenly dividing the range of values to create categories is also used by developers of the KBDI.

The KBDI is a more accurate measure than precipitation of the effect of weather factors on soil moisture, which is a direct influence on logging conditions. The KBDI is used by the National Forest Service to measure cumulative risk of drought and forest fires. It is a number representing the net effect of evapotranspiration (the rate of moisture use by vegetation) and precipitation in producing cumulative moisture deficiency in deep duff and upper soil layers (Keetch and Byrum, 1968). It was specifically designed to measure the potential of a forest fire and the KBDI number measures the amount of precipitation necessary to cause the soil to be saturated or muddy. The values of the index range from 0 to 800 units and represent a moisture reading ranging from 0 to 8 inches of water through the soil layer. At 8 inches of water, the KBDI assumes saturation. Zero is the point of no moisture deficiency and 800 is the maximum drought that is possible. At any point along the scale, the index number indicates the amount of net rainfall that is required to reduce the index to zero, or saturation (Keetch and Byrum, 1968). It is recorded at Remote Automated Weather Stations (RAWS) throughout the United States and used primarily in conjunction with the Wildland Fire Assessment System (WFAS). As a drought measure, it is essentially a measure of the soil moisture and thus we use it as such.

For the purpose of using the KBDI as a measure of soil moisture in the logging areas of the southeast, we could not use the reported KBDI output of the RAWS stations for three reasons. The RAWS stations are not located in all of the areas in the southeast that are heavily logged so our data would not capture significantly large logging areas (Figure 5 and 6). In addition, because the RAWS stations are automated, the readings

are not consistently reported due to various types of malfunctions. Lastly, many of the RAWS stations are considered to be second order stations, which means that the data from these stations does not undergo the frequent quality checks by National Weather Service employees that data from first order stations require.

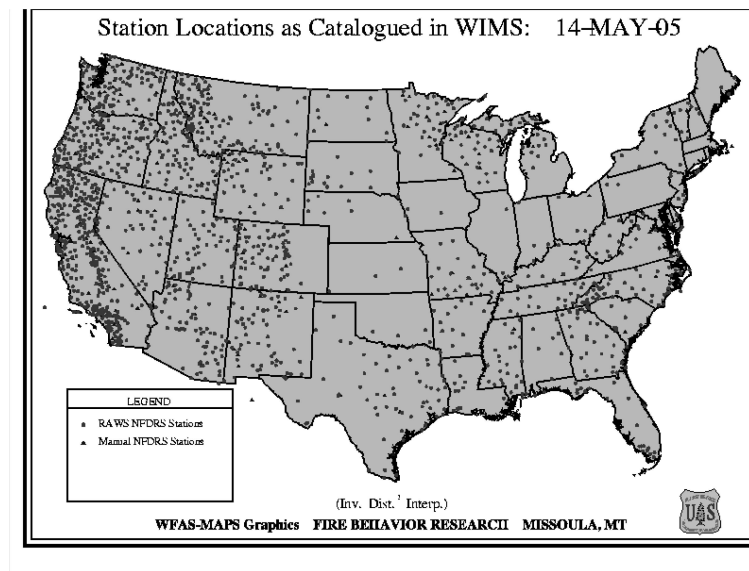


Figure 5: RAWS station locations [US Wildfire Assessment Service Web Site]

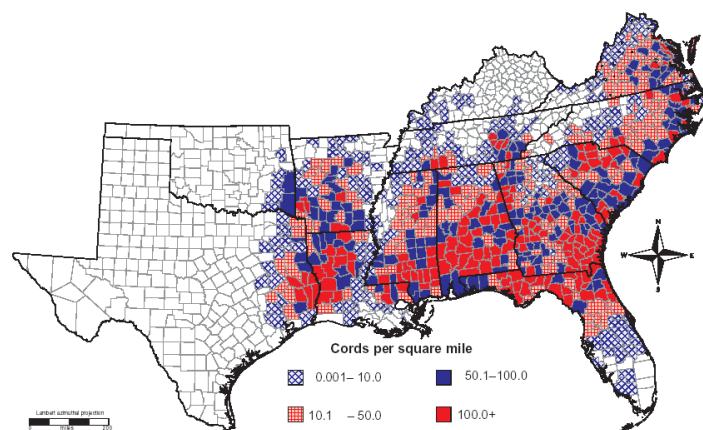


Figure 6: 2001 Softwood Pulpwood Production by State and Broad Species [Johnson and Steppeleton, 2003]

For these reasons, we calculated the KBDI values directly using the mathematical model

of Keetch and Byrum (1968). The inputs for the model were the precipitation and temperature readings collected from the 2004 database of the World Meteorological Organization (WMO) of which the National Weather Service (NWS) is a member. This database was accessed by way of the National Climatic Data Center (NCDC), which is the official source of weather data in the United States. The majority of the stations we selected to represent the southeast climates were first order stations.

KBDI is a differential formula, calculating subsequent KBDI as a function of current KBDI. Therefore, we did require the use of RAWS station output for an initial KBDI reading for each region as an input for the model. The formula for calculating the daily change in the KBDI is

$$dQ = \frac{(800 - Q)(0.968e^{0.0486T} - 8.30)dt}{1 + 10.88e^{-0.0441R}} * 10^{-3}$$

where Q is the last observance of the KBDI value (e.g., the previous day), the max daily temperature is denoted by T , the annual mean rainfall is denoted by R , and the number of days between observations is denoted by dt . The KBDI is normally calculated on a daily basis. Because it takes twenty-four hours for 0.20 inches of precipitation to evaporate, this value is subtracted from the daily precipitation if it exceeds 0.20 inches. The balance is subtracted from the KBDI value of the previous day and the calculated dQ is then added to produce the current KBDI value (Keetch and Byrum, 1968). The KBDI values in our data ranged from 117-587. The points above and below the median index value of 400 were classified as dry and wet, respectively. For simplification in the regression model, we label the points above the median with the binary digit one, to signify the higher KBDI value that corresponds to drier soil conditions. The points below the median are labeled zero and correspond to wet soil conditions. The expected and actual KBDI values for 2004 are located in Figure ??.

NCDC is an agency of the National Oceanic and Atmospheric Administration (NOAA), which also has a Climate Diagnostic Center (CDC). The CDC has put together a map which divides the United States on the county level into areas with similar historical climate readings such as precipitation and temperature. These areas are called climate divisions.

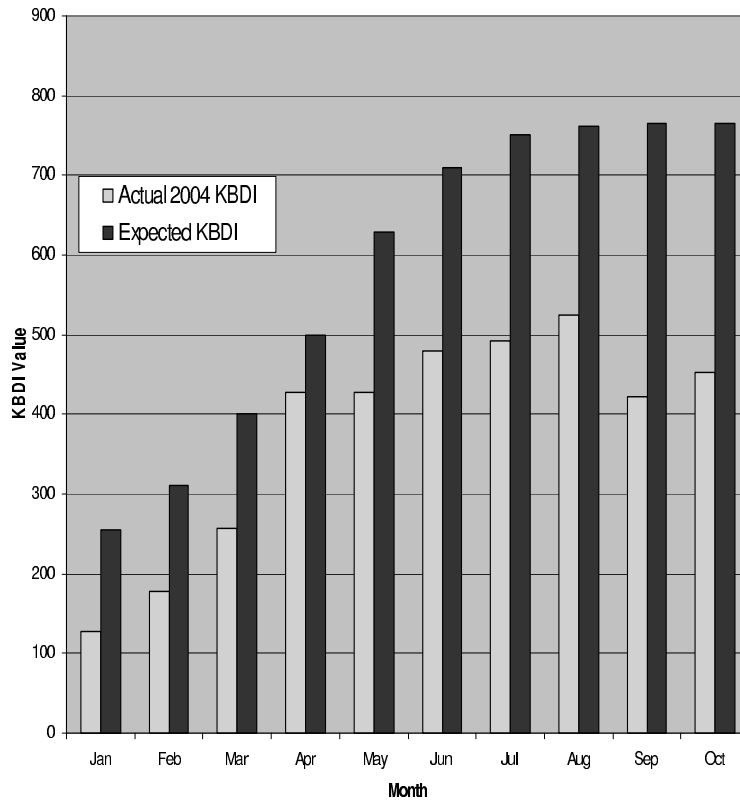


Figure 7: Expected and 2004 Actual Monthly KBDI values in the Southeast

Each state is divided into as many as eight climate divisions. Based on the climate divisions, we were able to assign the output of a weather station to the corresponding woodpulp productivity in 2001 (Figure 8).

We used the distribution of the logging areas as recorded in the 2001 survey of pulpwood production (Johnson and Steppleton, 2003). The areas of pulpwood production are fairly consistent with little change annually. A production change between years is typically less than 5% (Johnson and Steppleton, 2004).

When we consider the changes in the price and supply of timber with weather changes using our scaled data set, we found the estimates of the mean price and supply rainy and dry months to be statistically different with 95% level of confidence. The estimated mean

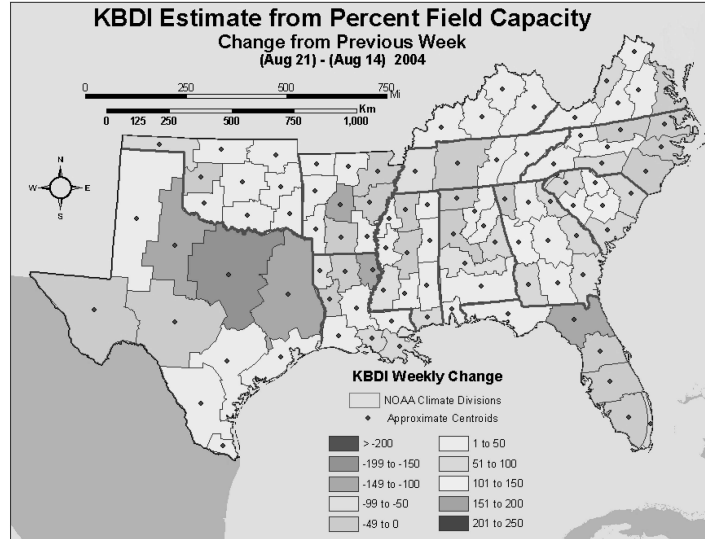


Figure 8: Climate Divisions in the South [US Forest Service Web Site]

Table 5: Comparison of Prices and Supply in Rainy and Dry Months

Classification	n	Mean	S.D.
Prices - Rainy	152	41.28	19.10
Prices - Dry	168	40.96	17.84
Supply - Rainy	102	1976	1971
Supply - Dry	110	1645	1645

Data has been uniformly scaled.

price and supply for both weather types were close with large standard deviations (see Table 5). The data was scaled to protect the confidentiality of the users of the Trading Center.

Using average rainfall data for 2004 from the National Weather Service, we divided the ten calendar months into rainy and dry months according to the expected and actual monthly precipitation falling above or below the median (Figure 9). The expected rainy months were March, June, July, August, and September. The actual rainy months in our data set were February, June, July, August and September. The expected and actual average rainfall in the southeast for these months was above the median of 4.54 and 4 inches, respectively. The expected dry months were January, February, April, May, and October. The actual dry months were January, March, April, May and October.

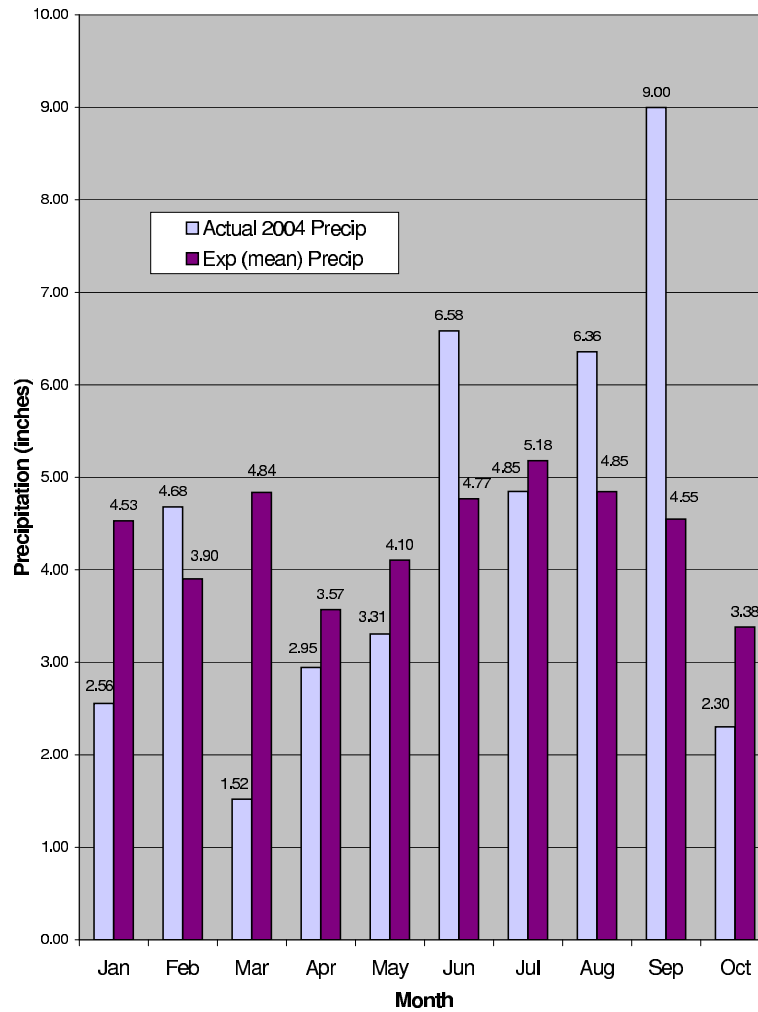


Figure 9: Expected and 2004 Actual Monthly Rainfall in the Southeast

4.2.2 Description of Good

An initial observation of the auction data revealed that the description of the good was related to other factors. Requests for timber that is cut to a custom length tended to have few bidders if any at all compared to auctions for timber of a standard length.

We found the estimates of the mean number of bidders for custom cut and standard length auctions to be statistically different with 95% level of confidence. The estimated mean number of bidders for the standard good was roughly quadruple that of the custom cut (see Table 6).

The description of the good was also related to whether there was a supply buildup over time (Figure 10). For example, we looked at the auctions of timber with custom descriptions

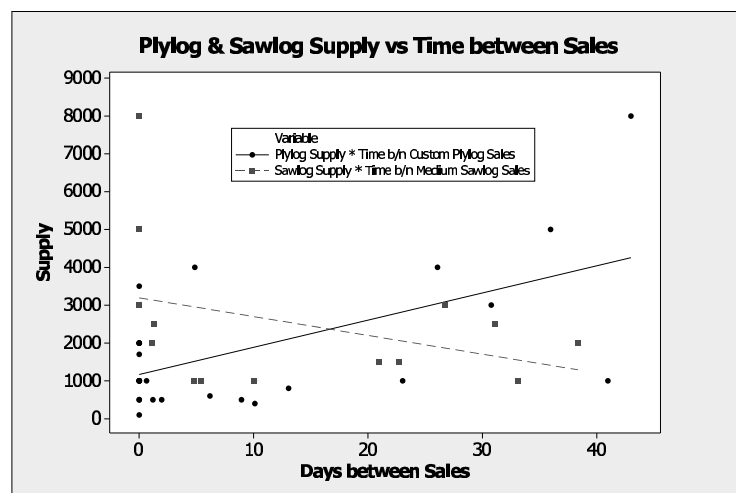
Table 6: Comparison of Bids for Custom and Standard Cuts of Timber

Classification	n	Mean	S.D.
Custom Bids	315	0.32	0.71
Standard Bids	192	1.24	1.27

Table 7: Sawlog and Plylog Supply vs. Time between auction closes

Classification	n	t-value	p-value	R-sq
Time b/n Sawlog Sales	14	-1.36	0.200	13.3%
Time b/n Plylog Sales	31	3.71	0.001	32.2%
Time b/n Standard Sales	139	0.79	0.429	0.5%
Time b/n Custom Sales	208	-0.09	0.932	0.0%

that had the greatest numbers of successful auctions. They were the auctions for custom cut plylogs and cut medium sawlogs. The successful timber auctions for plylogs tended to see an increase in the amount supplied as the time between the close of auctions increased. A simple regression analysis on the effect of time between sales of plylogs on the amount supplied showed a significant relationship with a p-value of 0.001. However, no significant relationship was found for successful auctions of sawlogs, which had a p-value of 0.2 (Table 7).

**Figure 10:** Scatterplot of Effect of Time between Sales on Plylog and Sawlog Online Supply

When we considered the effect of time between the close of auctions on standard and custom goods of all descriptions, we found there to be no significant relationship with a p-value of 0.429 for standard and a p-value of 0.932 for custom. Therefore, we did not include the time between auctions as a factor in the multi-variable regression model. Instead, we created a third category of description for plylogs because the successful auctions for plylogs represented about 40% of the successful custom auctions and aside from the other custom auctions, the supply of custom cut plylogs was related to the time between the close of the auctions.

Even if a supplier has the supply of timber and ability to cut it to order, she may not want to conduct the transaction to supply a custom order by online auction. This may be particularly true if the supplier does not regularly fill requests for custom orders.

There is less competition for custom orders and the price of custom orders tends to be higher than standard orders. In public auctions, when the item for auction is unique, the seller of the item initiates a forward auction. In B2B reverse auctions, where the seller cannot host a forward auction and the competition between them is lower, the seller and buyer would likely do better to negotiate over non-price attributes of the custom order than to participate in an auction.

4.2.3 Species of Timber

Hardwood and softwood are the two broad species of timber. The wood from broad-leaved trees such as oak is called hardwood and the wood from conifers such as pine is called softwood. Hardwood is known for its smooth fibers and is used in the production of bathroom and facial tissue and hardwood floors whereas softwood fibers are known for their strength and are useful in the production of paper bags, frames for homes, and corrugated boxes used in shipping.

In the southeastern United States, softwood trees grow and are logged in more abundance than hardwoods (Figures 6, 11 and 12). Softwood accounts for 60 – 70% of total Southern timber harvests (USDL, 1988). Therefore, the supply transacted in auctions in

which hardwood is requested is likely to be lower. Hardwood, in some cases may be requested as a substitute for softwood and a mix of the two woods may be used to make paper. An initial survey of the data revealed that 39% of the auctions initiated requested hardwood.

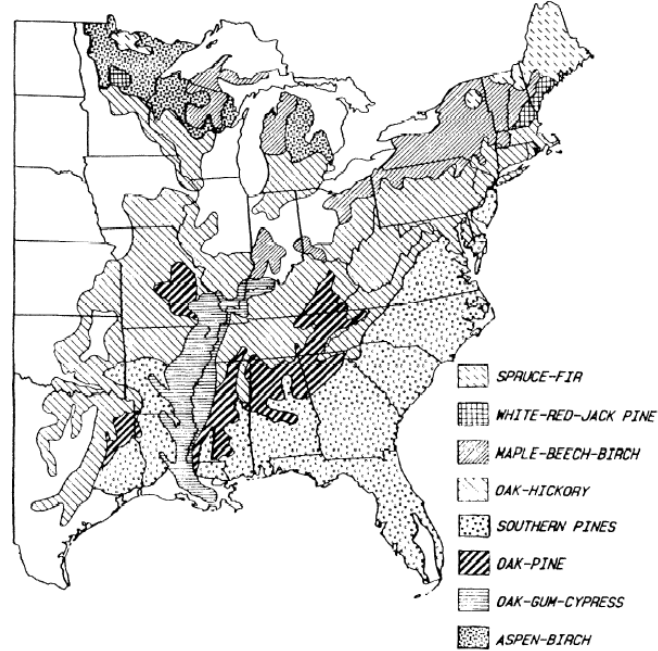


Figure 11: Major Forest Cover Types in the Southern United States [Eyre 1980]

4.2.4 Bidders

Empirical research on the relationship between the number of bidders and the success of procurement auctions shows that the number of bidders is influential in the competition witnessed in the auction (Millet et al, 2004). They suggest that subject to cost-benefit analysis, firms allocate resources to developing suppliers who will be high-quality bidders. We are interested in the significance of the number of bidders to the volume supplied online in our data to determine if mills need to consider developing bidders. In our data, we have an average of 1.24 bidders with a standard deviation of 1.2 and the number of bidders ranging from zero to eight. Accordingly, the majority of the auctions were not competitive and the full amount demanded was not supplied. This may be due to underdeveloped suppliers or another reason.

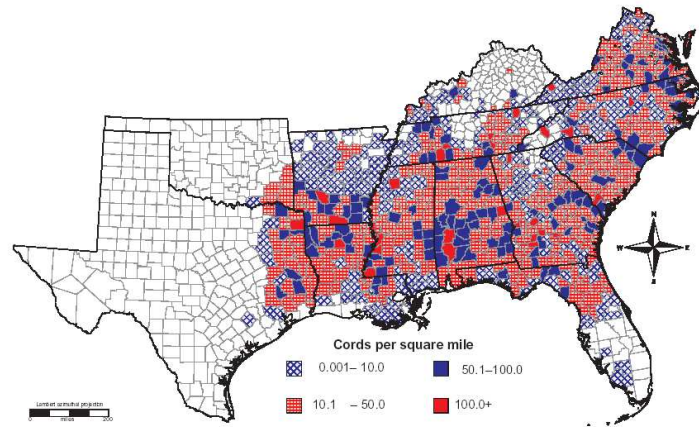


Figure 12: 2001 Hardwood Pulpwood Production by State and Broad Species [Johnson and Steppleton, 2003]

4.2.5 Demand and Reservation Price

As its demand increases, the mill desires a corresponding increase in the supply of timber. This desire might be indicated in the increase of the reservation price as the need of the mill for timber increases.

4.3 Regression Analysis

The goal of the regression analysis is to see how significant the internal and external factors were in influencing the volume of timber (Supply) transacted in online reverse auctions. The predictor variables we considered for analysis were the expected monthly precipitation, the actual monthly precipitation, the weighted average actual and expected soil moisture in the timber producing regions of the south on the closing day of the auction as measured using the KBDI index (KBDI), the expected KBDI in the Southeast (ExpKBDI), the volume of delivered timber requested (Demand), the categories for the number of bidders (BidCat), whether the description of the timber requested was for timber of standard length or for a custom cut (Descrp), the species of timber (Species), and the reservation price (Res) set by the auctioning mill. Both Demand and Supply are in units of tons. The variables for the expected and actual monthly precipitation, the species, the actual and expected KBDI, and the description of the requested tree were binary. The demand, supply, and reservation price

Table 8: Regression Variables

Variables	Description	Units
Supply	Cumulative supply from winning bidders	tons
Demand	Demand posted by the auctioneer	tons
BidCat	Category of number of bidders	0=zero bidders, 1=one or two, 2= more than two bidders
Res	Reservation price set by auctioneer	\$
Species	Species of tree requested	1=hardwood, 0=softwood
Descrp	Description of demand as listed by the auctioneer	1=custom cut or tree length 0=mixed hardwood or softwood
ExpPrecp	Expected monthly precipitation	1=wet, 0=dry
ActPrecp	Actual monthly precipitation	1=wet, 0=dry
KBDI	Keetch-Byrum Drought Index	1=dry, 0=wet
ExpKBDI	Expected Keetch-Byrum Drought Index	1=dry, 0=wet

were continuous quantitative variables. The number of bidders was categorical with a zero representing no bidders, one representing one or two bidders, and two representing more than two bidders. The variables, their descriptions and units of measure are summarized in Table 8. In running a stepwise regression using the Minitab software package, a p-value was set to 0.15 to enter and exit the model. The final regression model is as outlined in Table 9. There were 45 predictors and 507 complete observations.

4.4 Summary

4.4.1 Internal Factors

The description (Descrp) of the good had a significant relationship with the amount supplied. From our initial observation of the data, we found that 62% of the descriptions included a reference to the cut or length of the timber and represented 82% of the unsuccessful auctions so a relationship between description and volume supplied was expected. From the correlation matrix, in Table 11, which was also run using Minitab, we see that there is a negative correlation between the description of the good and the amount supplied. This indicates that custom goods tended to attract less supply.

The number of bidding suppliers sorted by category (BidCat) was also significantly related to the amount supplied and had the highest correlation with supply. This is expected

Table 9: Regression Model

Variables	β	t-value
Constant	-784.0	-3.7***
Descrp	592.6	2.96***
ExpPrecp	105.0	0.86
ActualPrecp	-205.9	-1.81*
ExpKBDI	171.2	1.59
Demand	0.13	3.97***
Res	13.32	3.13***
BidCat	391.5	2.07**
BidCat*Res	8.46	2.48***
ExpPrecp*BidCat	254	2.06**
ExpPrecp*Demand	-0.07	-1.86**
Demand*ExpKBDI	-0.12	-3.61***
Res*Descrp	-10.6	-2.99***
ActualPrecp*Demand	0.12	3.22***
Demand*Descrp	-0.05	-2.19**
Demand*BidCat	0.22	11.19***

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$.

R-sq = 71.9, R-sq(adj) = 71.0

Table 10: Descriptive Statistics

Variables	n	Min	Max	Mean	S.D.
Supply	507	0	8,000	755	1,471
Demand	507	1	20,000	2,317	2,464
BidCat	507	0	2	0.48	0.62
Res	507	0	96.63	45.37	20.59
Species	507	0	1	0.39	0.49
Descrp	507	0	1	0.62	0.49
ExpPrecp	507	0	1	0.41	0.49
ActPrecp	507	0	1	0.43	0.5
KBDI	507	1	0	0.47	0.5
ExpKBDI	507	1	0	0.44	0.49

Table 11: Correlation Matrix

Variables	Supply	Demand	BidCat	Res	Species	Descrp	ExpPrecp	ActPrecp	KBDI
Demand	0.513*								
BidCat	0.712*	0.315*							
Res	-0.016	-0.104	-0.109						
Species	0.053	0.158*	0.035	-0.518*					
Descrp	-0.162*	-0.128	-0.313*	0.613*	-0.035				
ExpPrecp	0.101	0.136	0.068	0.051	0.064	-0.025			
ActPrecp	0.101	0.116	0.069	-0.02	0.048	-0.084	0.275*		
KBDI	0.100	0.189*	0.134	-0.103	0.167*	-0.098	0.364*	0.326*	
ExpKBDI	0.117	0.198*	0.138	-0.122	0.171*	-0.094	0.316*	0.275*	0.927*

* $p < 0.001$

because if the auctions had no bidders then the amount supplied was certain to be zero. From our initial observation of the data, we found that 58% of the auctions did not attract any bidders so a strong relationship between bid category and volume supplied was expected. From the descriptive statistics in Table 10, we see that the mean bid category was between one and two. This indicates that on average the auctions had less than two total bidders.

We found that the bid category was significantly related to the description of the good. The negative correlation indicates that more suppliers were available to bid when the good for auction was standard timber than when it was for custom timber. Because there is a significant correlation between the bid category and the description, we conclude that before resources are allocated to develop suppliers, mills should be conservative in their expectations of the supply available, especially for custom goods.

The demand (Demand) for the good had a significant positive affect on the volume supplied. As more volume was demanded, more volume was supplied. This relationship is apparent in the correlation matrix in Table 11. We see that the demand was positively correlated with supply.

The supply was significantly related to the reservation price (Res). This is likely because of the significant price difference between the custom and standard timber. The reservation price had a small but negative correlation with supply. Because the reservation price was highly correlated with the description at a significant value of 0.613, a higher reservation price is most likely associated with custom goods, which tended to have fewer successful

auctions. So there is an indirect relationship between the reservation price and supply. This is confirmed by the significant interaction between the reservation price and the description, a detailed discussion of which is to follow.

There was a combined effect of demand and the bidder category which had a positive affect on the volume of supply. From Table 11, we see that demand and bidder category were positively correlated with each other as well as with supply. Therefore, an increase in either variable was related to an increase in supply. In addition, an increase in the volume demanded tended to be accompanied by an increase in the number of bidders. This indicates that large volumes of demand accompanied by a relatively large number of bidders were important factors in realizing high volumes of supply.

The bid category and the reservation price had a combined positive affect on the volume supplied. These two variables were negatively correlated indicating that an increase in the reservation price was associated with a decrease in the number of bidders. This is a counter-intuitive result because one would assume that a supplier would prefer to sell her goods at a higher price should a buyer make such an offer. The counter-intuitive result may be due to the high correlation that both variables have with the description of the timber. Timber that had a description that included terms such as 'custom cut' or 'tree length' tended to have a higher reservation price and a lower number of bidders. From our initial observations of the data we found 62% of the auctions had requests for customized timber. In addition, there was a combined effect of reservation price and description on the volume supplied. The coefficient was negative indicating that a higher reservation price for a custom good was likely to have less supply than otherwise. As a result, it is likely that the behavior of bidders toward customized timber has a greater affect on volume than how they would behave toward a standard good.

There was a significant negative effect of demand and description on the volume supplied. From the correlation matrix, we see that there is a significant negative correlation between demand and description, which indicates that as one variable increases the other decreases. An increase in the description variable means that the timber has a custom description and a further increase corresponds to description of custom cut plywood. This means that

the online demand for custom timber or plywood tended to be higher than the demand for standard wood. In addition, the volume of supply transacted online tended to decrease when the auction was for custom goods or plylogs.

The species (Species) was not found to have significant influence on the volume supplied. Given the lower amount of hardwood grown in the south, one would suspect the auctions for hardwood to be less successful than those for softwood. The fact that species was not significant in influencing supply indicates that the suppliers did not find it difficult to satisfy the hardwood demands for the lower volumes transacted in a spot market. Timber companies and mills may prefer to transact hardwood online because the softwood is logged for larger contracts and because hardwood can be added to softwood for the production of paper.

4.4.2 External Factors

Of the three indicators of seasonal patterns, only one proved to be independently significant in predicting the volume supplied online. The actual precipitation (ActualPrecp) had a significant negative relationship with the volume supplied. As the number of inches of actual precipitation increased, the amount of volume supplied decreased. This is intuitive as the precipitation is a factor in the ability of loggers to log and deliver timber.

Each indicator of seasonal patterns had combined affects on the amount supplied online. The actual precipitation and the demand had a combined affect on the volume supplied. They had a significant positive influence and they were positively correlated. Therefore, an increase in the actual precipitation and the volume demanded corresponded to an increase in the volume supplied. This result seems counter-intuitive because actual precipitation alone has a significant negative relationship with supply. But because demand alone has a significant positive relationship with supply and a strong correlation, the combined affect on supply was positive. This indicates that for larger demand, loggers are able to fill supply. This could be for a host of reasons including that loggers may be more willing to tap into all-weather stands of trees when the volume requested is large. These stands tend to be at higher elevations and are less influenced by precipitation compared to stands in the valley.

The expected KBDI (ExpKBDI) was more significant than the actual KBDI and the two are highly correlated. The expected KBDI and demand had a significant combined affect on the volume supplied. They were also significantly and positively correlated indicating that an increase in the expected KBDI value corresponded to an increase in the amount demanded. This means that an increase in the volume demanded and better logging conditions, measured by a decrease in soil moisture, corresponded to a decrease in the volume supplied online. This result is intuitive when we consider the fact that when the logging conditions are suitable, a mill is better off obtaining wood offline either by cutting from its own woodlands or by using the supply from its long-term contracts.

The expected precipitation interacted with both the demand and the number of bidders. The expected precipitation combined with each factor to have opposing affects on the volume supplied. Demand that was requested in a month that was expected to have higher precipitation, corresponded with a decrease in the volume supplied. In addition, an increase in the number of bidders during months that were expected to have higher levels of precipitation corresponded to an increase in the volume supplied. There was a significant positive correlation between expected precipitation and demand although no significant correlation was found with the bid category. It follows intuition that more volume be demanded online in periods that were expected to have more rain as mills want to make sure they have more than enough timber to avoid a shortage. To do so, they turn to the spot market. It also follows that the volume of supply increases when more loggers are available in periods of expected precipitation.

As we compare the effect of the expected and actual levels of precipitation each month and the expected and actual KBDI values on the volume supplied, we first consider the correlation between the values. There is a significant positive correlation between each of the measures of seasonal patterns. There is a less strong positive correlation between the precipitation and KBDI values which is most likely because the KBDI considers the effect of temperature and other variables on soil moisture in addition to precipitation. Precipitation in a winter month will likely not have the same affect on logging conditions as the same amount of precipitation in a summer month.

We next compare the effects of the interactions between the KBDI, the actual and expected precipitation, and the demand on supply. We find that when the month is expected to have high precipitation and the demand is increased, the supply decreases. Likewise, when there is more expected soil moisture as shown by a higher KBDI and an increased demand, the supply decreases. However, when the month actually does have high precipitation and demand is increased, the supply increases. This implies that the historical patterns of precipitation do not necessarily set the environment for the market and that the market is somewhat elastic with respect to the current logging conditions. It is possible that logging crews and long-term contracts are determined based on past weather patterns but current weather conditions have an affect on supply.

We also considered the independent effect of the time between auctions on the supply considering that suppliers might have time to gather supply between auctions. We found no significant relationship for goods with a standard or custom description (see Figure 13).

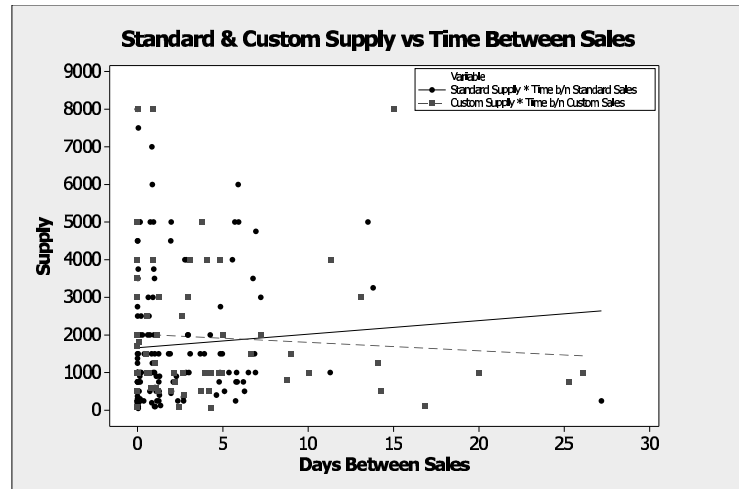


Figure 13: Scatterplot of Effect of Time between Sales on Standard and Custom Online Supply

4.4.3 Selected Pricing Rule

One effect of each auction rule on the price and quantity that a bidder submits is that a bidder has a stronger incentive to reduce the quantity portion of her bid under the uniform

rule than under the discriminatory rule (Tenorio, 1997). The idea is that the winners of the uniform rule pay the lowest accepted price for each awarded unit hence it is better for a bidder to reduce her quantity to compete with lower quantity bidders and let lower valuing bidders enter the set of winners. The private value multi-unit auction framework used in this exchange employs the discriminatory pricing rule. Therefore, we look for evidence of that suppliers do not reduce the quantity they supply in these auctions.

There were multiple winners and the winners were chosen based on price then volume. The discriminatory pricing rule is used so the price at which each supplier sells her volume is equal to the lowest bid that she placed. In the majority of the auctions, the winning bid prices were close, if not equal, to the opening price set by the auctioneer. This indicates that there was little competition over price because the volume request was not yet filled. The objective of the auction tended to be securing volume at a price suitable to the auctioning mill. With volume being the dominant factor in these auctions, a strategic supplier should not benefit from withholding the sale of volume if the prices in the high and low price season have been scaled to realize seasonal differences in cost.

The reservation price is a significant independent predictor of supply but it is not significantly correlated with the number of bidders nor with the demand or supply. This implies that a change in the reservation price will not attract or discourage the amount supplied via online auctions for timber. The reservation price is also not significantly correlated with the seasonal indicators of precipitation or KBDI values. In total, this implies that suppliers do not have a price-incentive to withhold supply in a dry month in favor of a better price in a rainy month.

The fact that the reservation price is not significantly correlated with several key internal and external factors nor with the volume supplied does not contradict the idea that the discriminatory auction that is implemented assists in the prevention of suppliers placing bids for a lower quantity than they have available as predicted by Tenorio (1997).

4.5 Industrial Implications

Based on this analysis, the low aggregation of timber supply online is most significantly associated with the description of the good, the volume demanded and the number of bidders. Successful auctions on the exchange were more often for standard goods than for commodities although some custom goods, such as plylogs were successfully transacted online.

The output of the model is useful in characterizing the supply transacted via the on-line auctions of the Trading Center exchange. Behavior outside of the exchange is likely influenced by a number of important external market factors that are not accounted for in the exchange data. For example, different auctioneer goals and auction rules, the collusion between suppliers, the size of suppliers, trends in the logging labor force, the influence of substitutes such as the recovered paper international market and the latest installation of satellite chip mills to name a few all affect the characterization of total supply.

Pulp and paper mills should gain a firm understanding of their procurement needs and determine whether the current auction design assists them in reaching their goal of accumulating supply. They should consider alternative outlets for procuring customized timber should they seek a higher success rate. Mills should consider these findings for dedicated analysis of their electronic procurement decision process and their spot market strategy in particular.

The number of bidding suppliers sorted by category (BidCat) had the largest independent influence on the amount supplied. It also had the highest correlation with supply. This indicates that the bidders were constrained in the amount of supply they could satisfy. Therefore to satisfy demand, mills should invite as many suppliers to bid as possible. This is withstanding qualitative preferences that may take precedence over volume even on the spot market but because all bidders must meet requirements to become members of the exchange, the qualitative preferences should not play a large role in auction behavior.

The relationship between the time between the close of auctions and the supply of timber was different depending on the description of the timber. For example, the time between the close of successful auctions for sawlogs did not have a significant relationship with the

volume of sawlogs supplied but there was a significant relationship with the volume of plylogs supplied. This suggests that the online marketplace is more successful in fulfilling requests for certain descriptions of timber. More work into the effectiveness of the marketplace in meeting specific demands could characterize in more detail the industry market behavior both offline and online.

4.6 Recommendations for Additional Data Collection

There has been much research activity concerning bidder behavior surrounding the timber that is auctioned from the U.S. Forest Service. The empirical research results were made possible because the outcome of the auction is public information. However, the outcome of auctions between logging companies and private landowners is not public. A third-party online auction host that is able to collect sufficient data on the auctions that it processes can serve its clients by recommending the auction design that will be most efficient in reaching client goals.

We were not able to incorporate all of the potential predictors in our model because of unavailability of data. As the value of e-commerce increases for procurement in the pulp and paper industry, additional factors should be considered for further study. Data on two specific factors would shed light on bidder behavior over time. The first factor is the geographic location of the mill and the tracts from which the timber is logged. This data would allow the study on behavior between mills and logging companies over time in the context of a sequential auction. For example, do the same suppliers tend to bid and are there certain types of auction requests that certain logging companies tend to fill?

The second factor is the number of the invited bidders. This second factor has been shown to be fairly significant and closely related to the number of bidders when the type of auction conducted allows for the current winning bid to be revealed to other bidders. The behavior of invited bidders has been studied because not all invited bidders submit bids even after they have accepted the invitation to participate in the auction. The percent of the invited bidders that actually place a bid was found to be the most significant factor after number of bids (Millet et al. 2004). Information on this second factor will help mills

determine a minimum number of bidders to purposefully invite to assure that all requested demand is filled and satisfy the goal of the auction. Although logic would dictate that to satisfy the goal of the auction auctioneers would invite all bidders to participate in every auction, the fact that businesses are selective in who they invite to bid in an auction gives evidence that they do want to control to whom they reveal their demand. Having a minimum number in mind may help them weigh the benefit of accomplishing their auction goal with the risk of revealing information about their demand.

4.7 Future Research

Some interesting extensions of this work might involve the development of auction models that consider resale opportunities in determining the auction outcome. A similar study was done for timber auctions from the U.S. Forest Service (Haile, 2001) and concluded that bidder valuations were higher when the value of the option to sell in the resale market was high and the option value to buy in the resale market was low.

Also, further study on the application of this model to other time periods of auction activity for timber procurement would help validate its usefulness as a predictive model for expected supply from an initiated auction.

We did not include the spot market supply for the raw material recovered fiber, which is a substitute for the natural wood fiber of timber. The pulp and paper industry has set the goal of increasing the use of recovered paper as an input by 60% by year 2006 (Pulp & Paper Weekly, 2004). When logging becomes difficult during wet-weather months, pulp and paper mills can turn to recovered fiber for input.

The spot market is used to handle fluctuations in demand in other commodity markets as well (Bichler, et al., 2002). Some examples include chemicals (ChemConnect.com) and electricity (Chonawee, S. et al., 2001). The external factor that is similar among agricultural commodity spot markets is that of seasonal patterns. Futures markets already incorporate the effect of weather on market performance. For commodities that are not agricultural products, external factors such as substitutes and business relationships may be important factors to gauge in the measurement of auction success. The design of the auction will

dictate how important the internal factors of the auction will be. For example, a sequential auction will have time sensitive factors to consider while a combinatorial auction will have preferences of bidder weights to take into account.

CHAPTER V

SUMMARY

In this dissertation, we have theoretically and empirically studied the influence of auction design on the outcome of the auction. The goals of the auction participants included maximizing expected return from the auction or maximizing auction success as measured by transaction volume. Apart from the theoretical results of sequential auction designs, we studied the implementation of a single auction design used in an online exchange for timber procurement. We found that apart from the auction design, factors that are either internal or external to the auction environment can affect the success of the auction participants in reaching their goals.

We developed a new auction design, the alternating reverse and forward auction, which allows buyers and suppliers to alternate auction roles of bidder and auctioneer in a sequential auction. We compared the performance of this new auction design with that of two conventional auction designs, the reverse and forward sequential auction. We also determined the optimal bid strategy for each market side.

We further developed the conditions under which each auction design would be a market equilibrium and when neither of the conventional auction designs would be an equilibrium. We found the alternating auction to be a viable alternative to holding no auction at all when neither the forward nor the reverse auction is a market equilibrium. We then studied the conditions for each auction design to be an equilibrium auction design for a specific case of up to twenty suppliers and periods. We found the reverse auction to be the most robust auction design preference of our PDSM models. Furthermore, in the test range, the RFA auction is an equilibrium when there are an even number of periods and at least three buyers.

We found that an upper bound exists on the number of periods in the RA and FA PDSM auction models when there are at least three buyers. There exists a conditional

upper bound on the number of periods in the RFA model. The benefit of an additional period to the expected return depends on the value of variables in most cases. We also determined the value to a strategic supplier of the option to learn her future valuation in a two period sequence, which is important because she is subject to a negative expected profit when initiating a forward auction in the second period.

Using data from a transaction processing network that hosts online business-to-business timber auctions and that was interested in increasing the number of successful auctions, we found plausible explanations for online auction performance. This work is the first work that we are aware of that focuses on the bidding activity between the logging company and the mill. Based on our analysis, the aggregation of timber supply online is most significantly associated with the description of the good, the volume demanded and the number of bidders. Some custom goods, such as plylogs were successfully transacted online, however, successful auctions on the exchange were more often for standard goods than for commodities. As the value of e-commerce increases for procurement in the pulp and paper industry, additional factors should be considered for further study.

The collection of additional data would enable researchers to empirically test predictive models of industry behavior. This would be especially beneficial if the online auction activity was coupled by a derivatives market for timber production, which may be an option given the current industry transition and projected increased dependence of mills on privately owned timber.

Both markets and individual auction participants use online auctions strategically. As they explore the options available through online auctions, they are able to understand what works well for their particular business environment and what does not.

The goal of this thesis was to investigate how online auctions can be used to achieve stability in markets. Specifically, we studied how the online auction mechanism has been used in an industrial setting and a new auction design to that may aid in increasing the adoption on online auctions by hesitant participants. This goal has been accomplished. We now understand that the alternating auction sequence is a viable option for online business-to-business auctions and in some cases is an equilibrium. We have also studied

bidder behavior in timber auctions between logging companies and mills for a plausible explanation for the outcome of low aggregation of supply in the online auctions. The auctions for timber between these two segments of the market had not previously been studied. These are contributions to the body of knowledge in the field of auction design.

APPENDIX A

PROOFS OF PROPOSITIONS 3.4.1-3.4.8

In all of the following proofs, the assumption of bidder symmetry in the independent private values model allows us to study an arbitrary buyer or supplier; all others will behave in the same way.

Proposition 3.4.1(RA Expected Supplier Profit and Bid Strategy) The optimal equilibrium bid function when competing in the N^{th} -to-last period of a PDSM reverse auction with S suppliers is

$$b_N = v_N + \frac{N-1}{S(S-N+1)}.$$

The expected profit to a supplier competing in the N^{th} -to-last period of a PDSM reverse auction with S suppliers competing is

$$P_N = \frac{(1-v_N)^S}{S} + \frac{N-1}{S(S-N+1)}.$$

Proof of Proposition 3.4.1

We first consider the last auction in the sequence and use N as our index for the purposes of induction (i.e., $N = 1$). Let $v_{(1)} \leq v_{(2)} \leq \dots \leq v_{(S)}$ be the order statistics of the S bidder types in the N^{th} -to-last period. Because the auctions we consider are sequential English auctions, for $N = 1$, the symmetric equilibrium strategy is to bid $b_1 = v_1$ (Milgrom and Weber, 2000)., where v_1 is the valuation in this period. Using integration-by-parts, the expected profit to the supplier is

$$\begin{aligned} P_1 &= \int_{v_1}^1 (v_{(2)} - v_1)(S-1)(1-v_{(2)})^{S-2} dv_{(2)} \\ &= - \left[(1-v_{(2)})^{S-1}(v_{(2)} - v_1) \right] \Big|_{v_1}^1 + \int_{v_1}^1 (1-v_{(2)})^{S-1} dv_{(2)} \\ &= \frac{(1-v_1)^S}{S}. \end{aligned}$$

Now we assume our induction hypothesis for the N^{th} -to-last period. Specifically, we assume that the optimal bid function is

$$b_N = v_N + \frac{N-1}{S(S-N+1)}$$

and the total expected profit over the final N periods of the auction sequence is

$$P_N = \frac{(1-v_N)^S}{S} + \frac{N-1}{S(S-N+1)}.$$

We now show that our bid function and the expected profit hold for the $(N+1)^{st}$ -to-last period. Note that in the $(N+1)^{st}$ -to-last period, v_N is not yet known, so we integrate over all possible values.

In the following equation, we substitute r for v_{N+1} in the limits of integration in order to derive the bid function by assuming that one strategic bidder of type v_{N+1} chooses to bid as type r instead. Although r is a function of v_{N+1} , for notational simplicity, we write r in the equation instead of writing it as a function of a private valuation. The necessary first order condition is determined with respect to r .

Note that because the winning supplier leaves after each auction period, having S suppliers in the N^{th} -to-last period implies that we will have $S+1$ suppliers in the $(N+1)^{st}$ -to-last period.

$$\begin{aligned} P_{N+1} &= \int_r^1 (b_{N+1}(v_{(2)}) - v_{N+1})((S+1)-1)(1-F(v_{(2)}))^{(S+1)-2} f(v_{(2)}) dv_{(2)} \quad (9) \\ &\quad + \int_0^r \left[\int_0^1 P_N(v_N) f(v_N) dv_N \right] ((S+1)-1)(1-F(v_{(2)}))^{(S+1)-2} f(v_{(2)}) dv_{(2)} \\ &= \int_r^1 (b_{N+1}(v_{(2)}) - v_{N+1})((S+1)-1)(1-F(v_{(2)}))^{(S+1)-2} f(v_{(2)}) dv_{(2)} \\ &\quad + \int_0^r \left[\int_0^1 \left(\frac{N-1}{((S+1)-1)((S+1)-N)} + \frac{(1-v_N)^{(S+1)-1}}{(S+1)-1} \right) f(v_N) dv_N \right] \\ &\quad * ((S+1)-1)(1-F(v_{(2)}))^{(S+1)-2} f(v_{(2)}) dv_{(2)}. \end{aligned}$$

We obtain the first order conditions using Leibnitz's rule where r is a function of v_{N+1} :

$$\begin{aligned}
\frac{dP_{N+1}}{dr} &= -(b_{N+1}(r) - v_{N+1}) * ((S+1) - 1)(1 - r)^{(S+1)-2} \\
&\quad + \left[\frac{N-1}{((S+1)-1)((S+1)-N)} + \frac{1}{(S+1)((S+1)-1)} \right] ((S+1) - 1)(1 - r)^{(S+1)-2} \equiv 0 \\
&= ((S+1) - 1)(1 - r)^{(S+1)-2} \left[\frac{N-1}{((S+1)-1)((S+1)-N)} + \frac{1}{(S+1)((S+1)-1)} \right. \\
&\quad \left. + v_{N+1} - b_{N+1}(r) \right] \\
&= ((S+1) - 1)(1 - r)^{(S+1)-2} \left[\frac{(S+1)N - (S+1) + (S+1) - N}{(S+1)((S+1)-1)((S+1)-N)} + v_{N+1} - b_{N+1}(r) \right] \\
&= ((S+1) - 1)(1 - r)^{(S+1)-2} \left[\frac{N((S+1) - 1)}{(S+1)((S+1)-1)((S+1)-N)} + v_{N+1} - b_{N+1}(r) \right] \\
&= ((S+1) - 1)(1 - r)^{(S+1)-2} \left[\frac{N}{(S+1)((S+1)-N)} + v_{N+1} - b_{N+1}(r) \right].
\end{aligned}$$

We now solve for the bid function:

$$b_{N+1}(r) = v_{N+1} + \frac{N}{(S+1)((S+1)-N)}.$$

Because the right hand side of the bid function does not depend on r ,

$$b_{N+1}(v_{N+1}) = v_{N+1} + \frac{N}{(S+1)((S+1)-N)}.$$

The derivative of the expected profit, $P'_{N+1} > (<)0$ when $b_{N+1}(v_{N+1}) < (>)v_{N+1} + \frac{N}{(S+1)((S+1)-N)}$. Therefore $b_{N+1}(v_{N+1})$ is the optimal equilibrium response for the bidder of type v_{N+1} in the $(N+1)^{st}$ -to-last auction period. The first term, v_{N+1} , is the bidder private valuation in the $(N+1)^{st}$ -to-last period. This bid function shows that bidders inflate their bids above their private valuation. The amount of bid inflation depends on the number of competing suppliers and the length of the auction sequence.

As expected, bids will be inflated more in longer sequences because the increased future opportunities make accepting a smaller profit in the current period less attractive. Similarly, a shorter sequence with the same number of bidders will have increased competition and cause bids to be inflated less, because there are fewer opportunities available.

We substitute this optimal bid function into (13). Because the strategic supplier will bid as her true type, v_{N+1} , we replace r with v_{N+1} in the limits of integration. We reduce

P_{N+1} as follows:

$$\begin{aligned}
P_{N+1} &= \int_{v_{N+1}}^1 (b_{N+1}(v_{(2)}) - v_{N+1})((S+1)-1)(1-F(v_{(2)}))^{(S+1)-2} f(v_{(2)}) dv_{(2)} \\
&\quad + \int_0^{v_{N+1}} \left[\int_0^1 \frac{N-1}{((S+1)-1)((S+1)-N)} + \frac{(1-v_N)^{(S+1)-1}}{(S+1)-1} f(v_N) dv_N \right] \\
&\quad * ((S+1)-1)(1-F(v_{(2)}))^{(S+1)-2} f(v_{(2)}) dv_{(2)} \\
&= \int_{v_{N+1}}^1 (v_{(2)} + \frac{N}{(S+1)((S+1)-N)} - v_{N+1})((S+1)-1)(1-v_{(2)})^{(S+1)-2} dv_{(2)} \\
&\quad + \int_0^{v_{N+1}} \left[\frac{N-1}{((S+1)-1)((S+1)-N)} + \frac{1}{(S+1)((S+1)-1)} \right] \\
&\quad * ((S+1)-1)(1-v_{(2)})^{(S+1)-2} dv_{(2)} \\
&= \int_{v_{N+1}}^1 (v_{(2)} + \frac{N}{(S+1)((S+1)-N)} - v_{N+1})((S+1)-1)(1-v_{(2)})^{(S+1)-2} dv_{(2)} \\
&\quad + \left[\frac{N}{(S+1)((S+1)-N)} \right] ((S+1)-1) \int_0^{v_{N+1}} (1-v_{(2)})^{(S+1)-2} dv_{(2)} \\
&= \int_{v_{N+1}}^1 (v_{(2)} + \frac{N}{(S+1)((S+1)-N)} - v_{N+1})((S+1)-1)(1-v_{(2)})^{(S+1)-2} dv_{(2)} \\
&\quad + \left[\frac{N}{(S+1)((S+1)-N)} \right] (1 - (1-v_{N+1})^{(S+1)-1}).
\end{aligned}$$

We continue using integration-by-parts as follows:

$$\begin{aligned}
&\int_{v_{N+1}}^1 (v_{(2)} + \frac{N}{(S+1)((S+1)-N)} - v_{N+1})((S+1)-1)(1-v_{(2)})^{(S+1)-2} dv_{(2)} \\
&= - \left[(1-v_{(2)})^{(S+1)-1} (v_{(2)} + \frac{N}{(S+1)((S+1)-N)} - v_{N+1}) \right] \Big|_{v_{N+1}}^1 + \int_{v_{N+1}}^1 (1-v_{(2)})^{(S+1)-1} dv_{(2)} \\
&= \frac{(1-v_{N+1})^{(S+1)}}{(S+1)} + \frac{N(1-v_{N+1})^{(S+1)-1}}{(S+1)((S+1)-N)}.
\end{aligned}$$

Therefore,

$$\begin{aligned}
P_{N+1} &= \frac{(1-v_{N+1})^{(S+1)}}{(S+1)} + \frac{N(1-v_{N+1})^{(S+1)-1}}{(S+1)(S+1-N)} + \frac{N(1-(1-v_{N+1})^{(S+1)-1})}{(S+1)(S+1-N)} \\
&= \frac{(1-v_{N+1})^{S+1}}{S+1} + \frac{N}{(S+1)(S+1-N)}.
\end{aligned}$$

■

This expected profit function is comprised of the expected profit from the current auction and the future expected profit.

The contribution to the expected profit from winning in the N^{th} -to-last period is

$$P_N^A = \frac{(1-v_N)^S}{S} + \frac{(N-1)(1-v_N)^{S-1}}{S(S-N+1)}.$$

The contribution to the expected profit of losing in the N^{th} – *to – last* period and winning in a subsequent period is

$$P_N^B = \frac{(N-1)(1 - (1 - v_N)^{S-1})}{S(S - N + 1)}.$$

Note that, as would be expected $P_N = P_N^A + P_N^B$.

Propositions 3.4.2 and 3.4.4(RFA Expected Supplier Profit and Bid Strategy) The expected profit to a supplier in the N^{th} -to-last period of a PDSM alternating auction sequence when N is odd is

$$P_N = \frac{\frac{B-1}{B+1} - v_N}{S} + \Upsilon_N$$

where $\Upsilon_N = \sum_{p=0}^{\frac{N-3}{2}} \left(\prod_{q=0}^p \left(1 - \frac{1}{S-2q} \right) \right) \left[\frac{1}{(S-2p)(S-2p-1)} + \left(\frac{B-3}{2(B+1)} \right) \left(\frac{1}{S-2p-2} \right) \right],$

and when N is an even number is

$$P_N = \frac{(1 - v_N)^S}{S} + \frac{B-3}{2(B+1)(S-1)} + \Delta_N$$

where $\Delta_N = \sum_{p=1}^{\frac{N-2}{2}} \left(\prod_{q=1}^p \left(1 - \frac{1}{S-2q+1} \right) \right) \left[\frac{1}{(S-2p)(S-2p+1)} + \left(\frac{B-3}{2(B+1)} \right) \left(\frac{1}{S-2p-1} \right) \right].$

When N is an even number (i.e., the supplier is a bidder), the optimal bid function is

$$b_N(v_N) = v_N + \frac{B-3}{2(B+1)(S-1)} + \Delta_N.$$

Proof of Propositions 3.4.2 and 3.4.4

Recall that we assume that every RFA sequence ends with a forward auction. We first assume that there is only one auction in the sequence. Since it is a forward auction, the expected profit is the same as when $N = 1$ in the sequence of forward auctions: $P_1 = \frac{\frac{B-1}{B+1} - v_1}{S}$.

The expected profit when $N = 2$ is

$$P_2 = \int_r^1 (b_2(v_{(2)}) - v_2)(S-1)(1 - F(v_{(2)}))^{S-2} f(v_{(2)}) dv_{(2)} \quad (10)$$

$$+ \left(\frac{1}{S-1} \right) \int_0^r \left(\int_0^1 w_1 B(B-1) F(w_1)^{B-2} (1 - F(w_1)) f(w_1) dw_1 - \int_0^1 v_1 f(v_1) dv_1 \right)$$

$$* (S-1)(1 - F(v_{(2)}))^{S-2} f(v_{(2)}) dv_{(2)}.$$

We obtain the first order conditions using Leibnitz's rule. Although r is a function of v_2 , for notational simplicity, we write r in the equation instead of writing it as a function of a private valuation, which yields

$$\begin{aligned}\frac{dP_2}{dr} &= -(b_2(r) - v_2)(S-1)(1-r)^{S-2} + \frac{B-3}{2(B+1)(S-1)}(S-1)(1-r)^{S-2} \\ &= (S-1)(1-r)^{S-2} \left[\frac{B-3}{2(B+1)(S-1)} + v_2 - b_2(r) \right] \equiv 0.\end{aligned}$$

We solve for $b_2(r)$, which yields

$$b_2(r) = v_2 + \frac{B-3}{2(B+1)(S-1)}.$$

Because the bid function does not depend on r ,

$$b_2(v_2) = v_2 + \frac{B-3}{2(B+1)(S-1)}.$$

which is positive for $B \geq 3$. The derivative of the expected profit, $P'_2(v_2) > (<) 0$ when $b_2(v_2) < (>) v_2 + \frac{B-3}{2(B+1)(S-1)}$. Therefore $b_2(v_2)$ is the optimal equilibrium response for the bidder of type v_2 in the second-to-last alternating auction period. This bid function shows that the suppliers inflate their bids above their private valuation. The amount of the bid inflation depends on the number of buyers and suppliers.

We substitute this optimal bid function into equation(14). Because a strategic supplier will bid as her true type, v_2 , we replace r with v_2 in the limits of integration. The total expected profit of the 2-period sequence is

$$\begin{aligned}P_2 &= \int_{v_2}^1 (b(v_{(2)}) - v_2)(S-1)(1-F(v_{(2)}))^{S-2} f(v_{(2)}) dv_{(2)} \\ &\quad + \left(\frac{1}{S-1} \right) \int_0^{v_2} \left(\int_0^1 w_1 B(B-1) F(w_1)^{B-2} (1-F(w_1)) f(w_1) dw_1 - \int_0^1 v_1 f(v_1) dv_1 \right) \\ &\quad * (S-1)(1-F(v_{(2)}))^{S-2} f(v_{(2)}) dv_{(2)} \\ &= \int_{v_2}^1 \left(v_{(2)} + \frac{B-3}{2(B+1)(S-1)} - v_2 \right) (S-1)(1-F(v_{(2)}))^{S-2} f(v_{(2)}) dv_{(2)} \quad (11)\end{aligned}$$

$$\begin{aligned}&\quad + \left(\frac{1}{S-1} \right) \int_0^{v_2} \left(\int_0^1 w_1 B(B-1) F(w_1)^{B-2} (1-F(w_1)) f(w_1) dw_1 - \int_0^1 v_1 f(v_1) dv_1 \right) \\ &\quad * (S-1)(1-F(v_{(2)}))^{S-2} f(v_{(2)}) dv_{(2)} \quad (12) \\ &= \frac{(1-v_2)^S}{S} + \frac{B-3}{2(B+1)(S-1)}\end{aligned}$$

where (15) is simplified using integration-by-parts as follows:

$$\begin{aligned}
& \int_{v_2}^1 (v_{(2)} + \frac{B-3}{2(B+1)(S-1)} - v_2)(S-1)(1-F(v_{(2)}))^{S-2} f(v_{(2)}) dv_{(2)} \\
&= \int_{v_2}^1 (v_{(2)} + \frac{B-3}{2(B+1)(S-1)} - v_2)(S-1)(1-v_{(2)})^{S-2} dv_{(2)} \\
&= - \left[(1-v_{(2)})^{S-1} (v_{(2)} + \frac{B-3}{2(B+1)(S-1)} - v_2) \right]_{v_2}^1 + \int_{v_2}^1 (1-v_{(2)})^{S-1} dv_{(2)} \\
&= \frac{(1-v_2)^S}{S} + \frac{(B-3)(1-v_2)^{S-1}}{2(B+1)(S-1)}.
\end{aligned}$$

and where (16) is simplified as follows:

$$\begin{aligned}
& \left(\frac{1}{S-1} \right) \int_0^{v_2} \left(\int_0^1 w_1 B(B-1) F(w_1)^{B-2} (1-F(w_1)) f(w_1) dw_1 - \int_0^1 v_1 f(v_1) dv_1 \right) \\
& \quad * (S-1)(1-F(v_{(2)}))^{S-2} f(v_{(2)}) dv_{(2)} \\
&= \left(\frac{1}{S-1} \right) \int_0^{v_2} \left(\int_0^1 (w_1^{B-1} - w_1^B) B(B-1) dw_1 - \frac{1}{2} \right) (S-1)(1-v_{(2)})^{S-2} dv_{(2)} \\
&= \left(\frac{1}{S-1} \right) \int_0^{v_2} \left(B(B-1) \left(\frac{w_1^B}{B} - \frac{w_1^{B+1}}{B+1} \right) \Big|_0^1 - \frac{1}{2} \right) (S-1)(1-v_{(2)})^{S-2} dv_{(2)} \\
&= \left(\frac{1}{S-1} \right) \int_0^{v_2} \left(B(B-1) \frac{1}{B(B+1)} - \frac{1}{2} \right) (S-1)(1-v_{(2)})^{S-2} dv_{(2)} \\
&= \left(\frac{1}{S-1} \right) \int_0^{v_2} \left(\frac{B-1}{B+1} - \frac{1}{2} \right) (S-1)(1-v_{(2)})^{S-2} dv_{(2)} \\
&= \left(\frac{1}{S-1} \right) \left(\frac{B-1}{B+1} - \frac{1}{2} \right) (S-1) \frac{-(1-v_{(2)})^{S-1}}{S-1} \Big|_0^{v_2} \\
&= \left(\frac{1}{S-1} \right) \left(\frac{B-1}{B+1} - \frac{1}{2} \right) (1 - (1-v_2)^{S-1}) \\
&= \left(\frac{B-3}{2(S-1)(B+1)} \right) (1 - (1-v_2)^{S-1}).
\end{aligned}$$

Hence the result holds for $N = 1$ and $N = 2$.

For the remaining sequences, we will assume that N is even and prove by induction the result when $N+1$ is odd. Then, we assume that N is odd and prove the result when $N+1$ is even.

Assume that the expected profit to a supplier participating in the N^{th} -to-last period of a PDSM alternating auction that begins with a reverse auction (i.e., N is even) is

$$P_N(v_N) = \frac{(1-v_N)^S}{S} + \frac{B-3}{2(B+1)(S-1)} + \Delta_N.$$

and the optimal bid function in the N^{th} -to-last period is

$$b_N(v_N) = v_N + \frac{B-3}{2(B+1)(S-1)} + \Delta_N,$$

$$\text{where } \Delta_N = \sum_{p=1}^{\frac{N-2}{2}} \left(\prod_{q=1}^p \left(1 - \frac{1}{S-2q+1} \right) \right) * \left[\frac{1}{(S-2p)(S-2p+1)} + \frac{B-3}{2(B+1)} * \frac{1}{S-2p-1} \right].$$

The total expected profit for an $N+1$ period sequence of alternating auctions that begins with a forward auction (i.e., $N+1$ is odd) is

$$P_{N+1} = \frac{1}{(S+1)} \left(\int_0^1 w_{N+1} B(B-1) F(w_{N+1})^{B-2} (1-F(w_{N+1})) f(w_{N+1}) dw_{N+1} - v_{N+1} \right) \quad (13)$$

$$+ \left(1 - \frac{1}{(S+1)} \right) \left[\int_0^1 \left[\int_{v_N}^1 (b_N(v_{(2)}) - v_N) ((S+1)-2)(1-F(v_{(2)}))^{(S+1)-3} f(v_{(2)}) dv_{(2)} \right. \right. \quad (14)$$

$$\left. + \int_0^{v_N} \left(\frac{1}{(S+1)-2} \left(\int_0^1 w_{N-1} B(B-1) (F(w_{N-1}))^{B-2} (1-F(w_{N-1})) f(w_{N-1}) dw_{N-1} \right. \right. \quad (15)$$

$$\left. - \int_0^1 v_{N-1} f(v_{N-1}) dv_{N-1} \right) + \Delta_N \left((S+1)-2)(1-F(v_{(2)}))^{(S+1)-3} f(v_{(2)}) dv_{(2)} \right] f(v_N) dv_N \Big]$$

$$= \frac{\frac{B-1}{B+1} - v_{N+1}}{(S+1)} + \left(1 - \frac{1}{(S+1)} \right) \left[\frac{1}{(S+1)((S+1)-1)} + \frac{B-3}{2(B+1)} * \frac{1}{(S+1)-2} + \Delta_N \right],$$

where (17) is simplified as follows:

$$\begin{aligned} &= \frac{1}{(S+1)} \left(\int_0^1 w_{N+1} B(B-1) F(w_{N+1})^{B-2} (1-F(w_{N+1})) f(w_{N+1}) dw_{N+1} - v_{N+1} \right) \\ &= \frac{B(B-1)}{(S+1)} \left(\int_0^1 (w_{N+1}^{B-1} - w_{N+1}^B) dw_{N+1} \right) - \frac{v_{N+1}}{(S+1)} \\ &= \frac{B(B-1)}{(S+1)} \left(\frac{1}{B} - \frac{1}{B+1} \right) - \frac{v_{N+1}}{(S+1)} \\ &= \frac{\frac{B-1}{B+1} - v_{N+1}}{(S+1)}. \end{aligned}$$

The expression in (19) simplifies as follows:

$$\begin{aligned} &\int_0^{v_N} \left(\frac{1}{(S+1)-2} \left(\int_0^1 w_{N-1} B(B-1) (F(w_{N-1}))^{B-2} (1-F(w_{N-1})) f(w_{N-1}) dw_{N-1} \right. \right. \\ &\quad \left. \left. - \int_0^1 v_{N-1} f(v_{N-1}) dv_{N-1} \right) + \Delta_N \right) ((S+1)-2)(1-F(v_{(2)}))^{(S+1)-3} f(v_{(2)}) dv_{(2)} \Big] f(v_N) dv_N \\ &= \int_0^{v_N} \left(\frac{1}{(S+1)-2} \left(B(B-1) \left(\frac{1}{B} + \frac{1}{B+1} \right) - \frac{1}{2} \right) + \Delta_N \right) \left(((S+1)-2)(1-v_{(2)})^{(S+1)-3} \right) dv_{(2)} \\ &= \int_0^{v_N} \left(\frac{B-3}{2(B+1)(S+1)-2} + \Delta_N \right) \left(((S+1)-2)(1-v_{(2)})^{(S+1)-3} \right) dv_{(2)} \\ &= \left(\frac{B-3}{2(B+1)((S+1)-2)} + \Delta_N \right) (1 - (1-v_2)^{(S+1)-2}). \end{aligned}$$

When we combine the expressions in (18) and (19), they simplify as follows:

$$\begin{aligned}
& \left(1 - \frac{1}{(S+1)}\right) \left[\int_0^1 \left[\int_{v_N}^1 (b_N(v_{(2)}) - v_N)((S+1)-2)(1-F(v_{(2)}))^{(S+1)-3} f(v_{(2)}) dv_{(2)} \right. \right. \\
& \quad \left. \left. + \left(\frac{B-3}{2(B+1)((S+1)-2)} + \Delta_N \right) \left(1 - (1-v_2)^{(S+1)-2} \right) \right] f(v_N) dv_N \right] \\
= & \left(1 - \frac{1}{(S+1)}\right) \left[\int_0^1 \left[\int_{v_N}^1 \left(v_{(2)} + \frac{B-3}{2(B+1)(S-1)} + \Delta_N - v_N \right) ((S+1)-2)(1-v_{(2)})^{(S+1)-3} dv_{(2)} \right. \right. \\
& \quad \left. \left. + \left(\frac{B-3}{2(B+1)((S+1)-2)} + \Delta_{N-1} \right) \left((1-v_2)^{(S+1)-2} \right) \right] dv_N \right] \\
= & \left(1 - \frac{1}{(S+1)}\right) \left[\int_0^1 \left[\frac{(1-v_N)^{(S+1)-1}}{(S+1)-1} + \left(\frac{B-3}{2(B+1)(S-1)} + \Delta_N \right) \left((1-v_N)^{(S+1)-2} \right) \right. \right. \\
& \quad \left. \left. + \left(\frac{B-3}{2(B+1)((S+1)-2)} + \Delta_N \right) \left(1 - (1-v_N)^{(S+1)-2} \right) dv_N \right] \right] \tag{16} \\
= & \left(1 - \frac{1}{(S+1)}\right) \left[\int_0^1 \left[\frac{(1-v_N)^{(S+1)-1}}{(S+1)-1} + \left(\frac{B-3}{2(B+1)(S-1)} + \Delta_N \right) \right] \right] \\
= & \left(1 - \frac{1}{(S+1)}\right) \left[\frac{1}{(S+1)((S+1)-1)} + \left(\frac{B-3}{2(B+1)(S-1)} + \Delta_N \right) \right],
\end{aligned}$$

where (20) is the result of using integration-by-parts on the previous step as follows:

$$\begin{aligned}
& \int_{v_N}^1 \left(v_{(2)} + \frac{B-3}{2(B+1)(S-1)} + \Delta_N - v_N \right) ((S+1)-2)(1-v_{(2)})^{(S+1)-3} dv_{(2)} \\
= & - \left[(1-v_{(2)})^{(S+1)-2} \left(v_{(2)} + \frac{B-3}{2(B+1)(S-1)} + \Delta_N - v_N \right) \right]_{v_N}^1 + \int_{v_N}^1 (1-v_{(2)})^{(S+1)-2} dv_{(2)} \\
= & \frac{(1-v_N)^{(S+1)-1}}{(S+1)-1} + \left(\frac{B-3}{2(B+1)(S-1)} + \Delta_N \right) \left((1-v_N)^{(S+1)-2} \right)
\end{aligned}$$

$$\begin{aligned}
P_{N+1} &= \frac{\frac{B-1}{B+1} - v_{N+1}}{(S+1)} + \left(1 - \frac{1}{(S+1)}\right) \left[\frac{1}{(S+1)((S+1)-1)} + \frac{B-3}{2(B+1)} * \frac{1}{(S+1)-2} + \Delta_N \right] \\
&= \frac{\frac{B-1}{B+1} - v_{N+1}}{(S+1)} + \left(1 - \frac{1}{(S+1)}\right) \left(\frac{1}{(S+1)((S+1)-1)} + \frac{B-3}{2(B+1)} * \frac{1}{(S+1)-2} \right) \\
&\quad + \left(1 - \frac{1}{(S+1)}\right) + \Delta_N \\
&= \frac{\frac{B-1}{B+1} - v_{N+1}}{(S+1)} + \left(1 - \frac{1}{(S+1)}\right) \left(\frac{1}{(S+1)((S+1)-1)} + \frac{B-3}{2(B+1)} * \frac{1}{(S+1)-2} \right) + \left(1 - \frac{1}{(S+1)}\right) \\
&\quad + \sum_{p=1}^{\frac{N-2}{2}} \left(\prod_{q=0}^p \left(1 - \frac{1}{(S+1)-2q}\right) \right) \\
&\quad * \left[\frac{1}{((S+1)-2p-1)((S+1)-2p)} + \frac{B-3}{2(B+1)} * \frac{1}{(S+1)-2p-2} \right] \\
&= \frac{\frac{B-1}{B+1} - v_{N+1}}{(S+1)} + \left(1 - \frac{1}{(S+1)}\right) \left(\frac{1}{(S+1)((S+1)-1)} + \frac{B-3}{2(B+1)} * \frac{1}{(S+1)-2} \right) \\
&\quad + \left(\left(1 - \frac{1}{S+1}\right) \left(1 - \frac{1}{(S+1)-2}\right) \right) \left(\frac{1}{((S+1)-3)((S+1)-2)} + \frac{B-3}{2(B+1)} * \frac{1}{(S+1)-4} \right) + \dots \\
&\quad + \left(\left(1 - \frac{1}{S+1}\right) \left(1 - \frac{1}{(S+1)-2}\right) \dots \left(1 - \frac{1}{(S+1)-N+2}\right) \right) \\
&\quad * \left(\frac{1}{((S+1)-N+1)((S+1)-N+2)} + \frac{B-3}{2(B+1)} * \frac{1}{(S+1)-N} \right) \\
&= \frac{\frac{B-1}{B+1} - v_{N+1}}{(S+1)} + \sum_{p=0}^{\frac{(N+1)-3}{2}} \left(\prod_{q=0}^p \left(1 - \frac{1}{(S+1)-2q}\right) \right) \\
&\quad * \left[\frac{1}{((S+1)-2p-1)((S+1)-2p)} + \frac{B-3}{2(B+1)} * \frac{1}{(S+1)-2p-2} \right].
\end{aligned}$$

Hence, for odd values of N , the result holds.

Now assume that when the expected profit to the supplier participating in the N^{th} -to-last period of a PDSM alternating auction sequence that begins with a forward auction (i.e., N is odd) is

$$\begin{aligned}
P_N &= \frac{\frac{B-1}{B+1} - v_N}{S} \\
&\quad + \sum_{p=0}^{\frac{N-3}{2}} \left(\prod_{q=0}^p \left(1 - \frac{1}{S-2q}\right) \right) \left[\frac{1}{(S-2p-1)(S-2p)} + \frac{B-3}{2(B+1)} * \frac{1}{S-2p-2} \right].
\end{aligned}$$

We derive the optimal bid function for the $(N+1)^{st}$ -to-last period as follows:

$$\begin{aligned}
P_{N+1} &= \int_r^1 (b_{N+1}(v_{(2)}) - v_{N+1}) * ((S+1)-1)(1-F(v_{(2)}))^{(S+1)-2} f(v_{(2)}) dv_{(2)} \\
&\quad + \int_0^r \left[\int_0^1 P_N f(v_N) dv_N \right] * ((S+1)-1)(1-F(v_{(2)}))^{(S+1)-2} f(v_{(2)}) dv_{(2)}
\end{aligned}$$

We obtain the first order conditions using Leibnitz's rule. Although r is a function of v_{N+1} , for notational simplicity, we write r in the equation instead of writing it as a function of a

private valuation, which yields

$$\begin{aligned}\frac{dP_{N+1}}{dr} &= -(b_{N+1}(r) - v_{N+1})((S+1) - 1)(1-r)^{(S+1)-2} \\ &\quad + \left[\int_0^1 P_N(v_N) f(v_N) dv_N \right] ((S+1) - 1)(1-r)^{(S+1)-2} \equiv 0.\end{aligned}$$

We solve for $b_{N+1}(r)$, which is

$$b_{N+1}(r) = v_{N+1} + \int_0^1 P_N f(v_N) dv_N.$$

The right hand side of the bid function does not depend on r , so

$$\begin{aligned}b_{N+1}(v_{N+1}) &= v_{N+1} + \int_0^1 P_N f(v_N) dv_N \\ &= v_{N+1} + \int_0^1 \frac{\frac{B-1}{B+1} - v_N}{S} \\ &\quad + \sum_{p=0}^{\frac{N-3}{2}} \left(\prod_{q=0}^p \left(1 - \frac{1}{S-2q} \right) \right) \left[\frac{1}{(S-2p-1)(S-2p)} + \frac{B-3}{2(B+1)} * \frac{1}{S-2p-2} \right] f(v_N) dv_N \\ &= v_{N+1} + \int_0^1 \frac{\frac{B-1}{B+1} - v_N}{S} \\ &\quad + \sum_{p=0}^{\frac{N-3}{2}} \left(\prod_{q=0}^p \left(1 - \frac{1}{(S+1)-2q-1} \right) \right) \\ &\quad * \left[\frac{1}{((S+1)-2p-1)((S+1)-2p-2)} + \frac{B-3}{2(B+1)} * \frac{1}{(S+1)-2p-3} \right] f(v_N) dv_N \\ &= v_{N+1} + \int_0^1 \frac{\frac{B-1}{B+1} - v_N}{((S+1)-1)} f(v_N) dv_N + \Upsilon_{N+1} \\ &= v_{N+1} + \frac{\frac{B-1}{B+1} - \frac{1}{2}}{((S+1)-1)} + \Upsilon_{N+1} \\ &= v_{N+1} + \frac{B-3}{2(B+1)((S+1)-1)} + \Upsilon_{N+1},\end{aligned}$$

where

$$\begin{aligned}\Upsilon_{N+1} &= \sum_{p=0}^{\frac{N-3}{2}} \left(\prod_{q=0}^p \left(1 - \frac{1}{(S+1)-2q-1} \right) \right) \\ &\quad * \left[\frac{1}{((S+1)-2p-1)((S+1)-2p-2)} + \frac{B-3}{2(B+1)} * \frac{1}{(S+1)-2p-3} \right].\end{aligned}$$

The bid function is positive when $B \geq 3$. The derivative of the expected profit, $P'_{N+1} > (<) 0$ when $b_{N+1}(v_{N+1}) < (>) v_{N+1} + \frac{B-3}{2(B+1)((S+1)-1)} + \Upsilon_{N+1}$. Therefore $b_{N+1}(v_{N+1})$ is the optimal equilibrium response for the bidder of type v_{N+1} in the first of an alternating sequence.

The total expected profit function is

$$P_{N+1} = \int_{v_{N+1}}^1 (b(v_{(2)}) - v_{N+1}) * ((S+1) - 1)(1 - F(v_{(2)}))^{(S+1)-2} f(v_{(2)}) dv_{(2)} \quad (17)$$

$$+ \int_0^{v_{N+1}} \left[\int_0^1 P_N(v_N) f(v_N) dv_N \right] * ((S+1) - 1)(1 - F(v_{(2)}))^{(S+1)-2} f(v_{(2)}) dv_{(2)}.$$

We substitute the optimal bid function into equation (21). Because a strategic supplier will bid as her true type, v_{N+1} , when using the optimal bid function, we replace r with v_{N+1} in the limits of integration. Using integration-by-parts, we obtain

$$\begin{aligned} P_{N+1} &= \int_{v_{N+1}}^1 (b(v_{(2)}) - v_{N+1}) * ((S+1) - 1)(1 - F(v_{(2)}))^{(S+1)-2} f(v_{(2)}) dv_{(2)} \\ &\quad + \int_0^{v_{N+1}} \left[\int_0^1 P_N(v_N) f(v_N) dv_N \right] * ((S+1) - 1)(1 - F(v_{(2)}))^{(S+1)-2} f(v_{(2)}) dv_{(2)} \\ &= \int_{v_{N+1}}^1 \left(v_{(2)} + \frac{B-3}{2(B+1)((S+1)-1)} + \Upsilon_{N+1} - v_{N+1} \right) * ((S+1) - 1)(1 - v_{(2)})^{(S+1)-2} dv_{(2)} \\ &\quad + \int_0^{v_{N+1}} \left[\frac{B-3}{2(B+1)((S+1)-1)} + \Upsilon_{N+1} \right] * ((S+1) - 1)(1 - v_{(2)})^{(S+1)-2} dv_{(2)} \\ &= - \left[(1 - v_{(2)})^{(S+1)-1} \left(v_{(2)} + \frac{B-3}{2(B+1)((S+1)-1)} + \Upsilon_{N+1} - v_{N+1} \right) \right] \Big|_{v_{N+1}}^1 \\ &\quad + \int_{v_{N+1}}^1 (1 - v_{(2)})^{(S+1)-1} dv_{(2)} \\ &\quad + \left[\left(\frac{B-3}{2(B+1)((S+1)-1)} + \Upsilon_{N+1} \right) \left(1 - (1 - v_{N+1})^{(S+1)-1} \right) \right] \\ &= (1 - v_{N+1})^{S+1} + \left[(1 - v_{N+1})^{(S+1)-1} \left(\frac{B-3}{2(B+1)((S+1)-1)} + \Upsilon_{N+1} \right) \right. \\ &\quad \left. + \left(\frac{B-3}{2(B+1)((S+1)-1)} + \Upsilon_{N+1} \right) \left(1 - (1 - v_{N+1})^{(S+1)-1} \right) \right] \\ &= (1 - v_{N+1})^{S+1} + \frac{B-3}{2(B+1)((S+1)-1)} + \Upsilon_{N+1}. \end{aligned}$$

Hence for an even value of N , the result holds. ■

Proposition 3.4.3(FA Expected Supplier Profit) The expected profit to a supplier competing in the N^{th} -to-last period of a PDSM forward auction sequence with B buyers and S suppliers is

$$P_N = \frac{\frac{B-1}{B+1} - v_N}{S} + \sum_{p=0}^{N-2} \left(\prod_{q=0}^p \left(1 - \frac{1}{S-q} \right) \right) \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S-p-1}.$$

Proof of Proposition 3.4.3

We consider the last auction in the sequence (i.e., $N = 1$). The expected profit to the

supplier is given by

$$\begin{aligned}
P_1 &= \frac{1}{S} \left(\int_0^1 w_1 B(B-1) F(w_1)^{B-2} (1 - F(w_1)) f(w_1) dw_1 - v_1 \right) \\
&= \frac{B(B-1)}{S} \left(\int_0^1 (w_1^{B-1} - w_1^B) dw_1 \right) - \left(\frac{v_1}{S} \right) \\
&= \frac{B(B-1)}{S} \left[\frac{w_1^B}{B} - \frac{w_1^{B+1}}{B+1} \right] \Big|_0^1 - \left(\frac{v_1}{S} \right) \\
&= \frac{B(B-1)}{S} \left[\frac{1}{B} - \frac{1}{B+1} \right] - \left(\frac{v_1}{S} \right) \\
&= \frac{\frac{B-1}{B+1} - v_1}{S}.
\end{aligned}$$

Assume the expected profit to the supplier in the N^{th} -to-last period of a PDSM forward auction is

$$P_N = \frac{\frac{B-1}{B+1} - v_N}{S} + \sum_{p=0}^{N-2} \left(\prod_{q=0}^p \left(1 - \frac{1}{S-q} \right) \right) \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S-p-1}.$$

We now consider a supplier's expected profit if she is competing in the $(N+1)^{st}$ -to-last period of the auction.

$$\begin{aligned}
P_{N+1} &= \frac{1}{(S+1)} \left(\int_0^1 w_{N+1} B(B-1) F(w_{N+1})^{B-2} (1 - F(w_{N+1})) f(w_{N+1}) dw_{N+1} - v_{N+1} \right) \\
&\quad + \left(1 - \frac{1}{(S+1)} \right) \left(\int_0^1 P_N(v_N) f(v_N) dv_N \right) \\
&= \frac{B(B-1)}{(S+1)} \left(\int_0^1 (w_{N+1}^{B-1} - w_{N+1}^B) dw_{N+1} \right) - \frac{v_{N+1}}{(S+1)} \\
&\quad + \left(1 - \frac{1}{(S+1)} \right) \left(\int_0^1 \frac{\frac{B-1}{B+1} - v_N}{(S+1)-1} f(v_N) dv_N + \sum_{p=0}^{N-2} \left(\prod_{q=0}^p \left(1 - \frac{1}{(S+1)-1-q} \right) \right) \frac{\frac{B-1}{B+1} - \frac{1}{2}}{(S+1)-1-p-1} \right) \\
&= \frac{B(B-1)}{(S+1)} \left(\frac{1}{B} - \frac{1}{B+1} \right) - \frac{v_{N+1}}{(S+1)} \\
&\quad + \left(1 - \frac{1}{(S+1)} \right) \left(\int_0^1 \frac{\frac{B-1}{B+1} - v_N}{(S+1)-1} dv_N + \sum_{p=0}^{N-2} \left(\prod_{q=0}^p \left(1 - \frac{1}{(S+1)-1-q} \right) \right) \frac{\frac{B-1}{B+1} - \frac{1}{2}}{(S+1)-1-p-1} \right) \\
&= \frac{\frac{B-1}{B+1} - v_{N+1}}{(S+1)} + \left(1 - \frac{1}{(S+1)} \right) \left(\frac{\frac{B-1}{B+1} - \frac{1}{2}}{(S+1)-1} + \sum_{p=0}^{N-2} \left(\prod_{q=0}^p \left(1 - \frac{1}{(S+1)-1-q} \right) \right) \frac{\frac{B-1}{B+1} - \frac{1}{2}}{(S+1)-p-2} \right) \\
&= \frac{\frac{B-1}{B+1} - v_{N+1}}{(S+1)} + \left(1 - \frac{1}{(S+1)} \right) \left(\frac{\frac{B-1}{B+1} - \frac{1}{2}}{(S+1)-1} \right) + \left(\left(1 - \frac{1}{(S+1)} \right) \left(1 - \frac{1}{(S+1)-1} \right) \right) \left(\frac{\frac{B-1}{B+1} - \frac{1}{2}}{(S+1)-2} \right) + \dots \\
&\quad + \left(\left(1 - \frac{1}{(S+1)} \right) \left(1 - \frac{1}{(S+1)-1} \right) \dots \left(1 - \frac{1}{(S+1)-1-N+2} \right) \right) \left(\frac{\frac{B-1}{B+1} - \frac{1}{2}}{(S+1)-N+2-2} \right) \\
&= \frac{\frac{B-1}{B+1} - v_{N+1}}{(S+1)} + \sum_{p=0}^{N-1} \left(\prod_{q=0}^p \left(1 - \frac{1}{(S+1)-q} \right) \right) \frac{\frac{B-1}{B+1} - \frac{1}{2}}{(S+1)-p-1},
\end{aligned}$$

where the first term, $\frac{\frac{B-1}{B+1} - v_{N+1}}{(S+1)}$ is the expected profit from hosting the $(N+1)^{st}$ -to-last period and the summation term is the future expected profit if the supplier is not able to

host the forward auction in the first period but is able to host an auction in one of the future periods. ■

Proposition 3.4.5(RA Expected Buyer Surplus) The expected surplus to a buyer in N^{th} -to-last period of a PDSM reverse auction sequence with B buyers and S suppliers is

$$\begin{aligned} D_N &= \frac{w_N - \frac{2}{S+1}}{B} + \sum_{p=1}^{N-1} \frac{\frac{1}{2} - \frac{2}{S-p+1}}{B}, \text{ if } N > 1 \\ &= \frac{w_N - \frac{2}{S+1}}{B}, \text{ if } N = 1. \end{aligned}$$

Proof of Proposition 3.4.5

We first consider the last auction in the sequence (i.e., $N = 1$). Because this is the last period, each supplier will bid her valuation. Assuming the buyer who initiates the auction is randomly selected from among all buyers, the expected surplus to each buyer is

$$\begin{aligned} D_1 &= \frac{1}{B} \int_0^1 (w_1 - v_{(2)}) S(S-1) (1 - F(v_{(2)}))^{S-2} F(v_{(2)}) f(v_{(2)}) dv_{(2)} \\ &= \frac{S(S-1)}{B} \int_0^1 (w_1 v_{(2)} - v_{(2)}^2) (1 - v_{(2)})^{S-2} dv_{(2)} \\ &= \frac{S(S-1)}{B} \int_0^1 (w_1 - 2v_{(2)}) \frac{(1 - v_{(2)})^{S-1}}{S-1} dv_{(2)} \end{aligned} \tag{18}$$

$$\begin{aligned} &= \frac{S(S-1)}{B} \left[\frac{w_1}{S(S-1)} + \frac{2(1 - v_{(2)})^{S+1}}{S(S-1)(S+1)} \Big|_0^1 \right] \\ &= \frac{S(S-1)}{B} \left[\frac{w_1}{S(S-1)} - \frac{2}{S(S-1)(S+1)} \right] \\ &= \frac{w_1 - \frac{2}{S+1}}{B}, \end{aligned} \tag{19}$$

where (22) is from the integration-by-parts of the preceding line as follows:

$$\begin{aligned} &\int_0^1 (w_1 v_{(2)} - v_{(2)}^2) (1 - v_{(2)})^{S-2} dv_{(2)} \\ &= (w_1 v_{(2)} - v_{(2)}^2) \frac{-(1 - v_{(2)})^{S-1}}{S-1} \Big|_0^1 + \int_0^1 (w_1 - 2v_{(2)}) \frac{(1 - v_{(2)})^{S-1}}{S-1} dv_{(2)} \\ &= \int_0^1 (w_1 - 2v_{(2)}) \frac{(1 - v_{(2)})^{S-1}}{S-1} dv_{(2)}, \end{aligned}$$

and the term (23) is from the Integration by Parts of the preceding line as follows:

$$\begin{aligned}
& \int_0^1 (w_1 - 2v_{(2)}) \frac{(1 - v_{(2)})^{S-1}}{S-1} dv_{(2)} \\
&= (w_1 - 2v_{(2)}) \frac{(-(1 - v_{(2)})^S)}{S(S-1)} \Big|_0^1 - \int_0^1 \frac{2(1 - v_{(2)})^S}{S(S-1)} dv_{(2)} \\
&= \frac{w_1}{S(S-1)} + \frac{2(1 - v_{(2)})^{S+1}}{S(S-1)(S+1)} \Big|_0^1.
\end{aligned}$$

The numerator is the difference between the buyer's valuation and the expected second lowest bid. The denominator accounts for the random chance that the buyer is able to initiate the auction.

Assume the expected surplus to the buyer for a PDSM reverse auction with N periods remaining is

$$\begin{aligned}
D_N &= \frac{w_N - \frac{2}{S+1}}{B} + \sum_{p=1}^{N-1} \frac{\frac{1}{2} - \frac{2}{S-p+1}}{B}, \text{ if } N > 1 \\
&= \frac{w_N - \frac{2}{S+1}}{B}, \text{ if } N = 1.
\end{aligned}$$

We now consider the buyer's expected surplus at the $(N+1)^{st}$ -to-last period.

$$\begin{aligned}
D_{N+1} &= \frac{1}{B} \left(\int_0^1 (w_{N+1} - v_{N+1})(S+1)((S+1)-1)(1 - F(v_{N+1}))^{(S+1)-2} F(v_{N+1}) f(v_{N+1}) dv_{N+1} \right. \\
&\quad \left. + \int_0^1 D_N f(w_N) dw_N \right) \\
&= \frac{1}{B} \left(\int_0^1 (w_{N+1} - v_{N+1})(S+1)((S+1)-1)(1 - v_{N+1})^{(S+1)-2} v_{N+1} dv_{N+1} \right. \\
&\quad \left. + \int_0^1 \frac{w_N - \frac{2}{(S+1)}}{B} f(w_N) dw_N + \sum_{p=1}^{N-1} \frac{\frac{1}{2} - \frac{2}{(S+1)-p}}{B} \right) \\
&= \frac{(S+1)((S+1)-1)}{B} \int_0^1 (w_{N+1} - 2v_{N+1}) \frac{(1 - v_{N+1})^{(S+1)-1}}{(S+1)-1} dv_{N+1} \tag{20} \\
&\quad + \int_0^1 \frac{w_N - \frac{2}{(S+1)}}{B} f(w_N) dw_N + \sum_{p=1}^{N-1} \frac{\frac{1}{2} - \frac{2}{(S+1)-p}}{B}
\end{aligned}$$

$$\begin{aligned}
&= \frac{(S+1)((S+1)-1)}{B} \left[\frac{w_{N+1}}{(S+1)((S+1)-1)} + \frac{2(1 - v_{N+1})^{(S+1)+1}}{(S+1)((S+1)-1)((S+1)+1)} \Big|_0^1 \right] \tag{21} \\
&\quad + \int_0^1 \frac{w_N - \frac{2}{(S+1)}}{B} f(w_N) dw_N + \sum_{p=1}^{N-1} \frac{\frac{1}{2} - \frac{2}{(S+1)-p}}{B} \\
&= \frac{w_{N+1} - \frac{2}{(S+1)+1}}{B} + \frac{1}{2B} - \frac{2}{(S+1)B} + \sum_{p=1}^{N-1} \frac{\frac{1}{2} - \frac{2}{(S+1)-p}}{B} \\
&= \frac{w_{N+1} - \frac{2}{(S+1)+1}}{B} + \frac{\left(\frac{1}{2} - \frac{2}{S+1}\right)}{B} + \frac{\left(\frac{1}{2} - \frac{2}{(S+1)-1}\right)}{B} + \dots + \frac{\left(\frac{1}{2} - \frac{2}{(S+1)-N+1}\right)}{B} \\
&= \frac{w_{N+1} - \frac{2}{(S+1)+1}}{B} + \sum_{p=1}^N \frac{\frac{1}{2} - \frac{2}{(S+1)-p+1}}{B}
\end{aligned}$$

where (24) is from integration-by-parts of the preceding line as follows:

$$\begin{aligned}
& \int_0^1 (w_{N+1}v_{N+1} - v_{N+1}^2)(1 - v_{N+1})^{(S+1)-2} dv_{N+1} \\
&= (w_{N+1}v_{N+1} - v_{N+1}^2) \frac{-(1 - v_{N+1})^{(S+1)-1}}{(S+1) - 1} \Big|_0^1 + \int_0^1 (w_{N+1} - 2v_{N+1}) \frac{(1 - v_{N+1})^{(S+1)-1}}{(S+1) - 1} dv_{N+1} \\
&= \int_0^1 (w_{N+1} - 2v_{N+1}) \frac{(1 - v_{N+1})^{(S+1)-1}}{(S+1) - 1} dv_{N+1},
\end{aligned}$$

and the term (25) is from integration-by-parts of the preceding line as follows:

$$\begin{aligned}
& \int_0^1 (w_{N+1} - 2v_{N+1}) \frac{(1 - v_{N+1})^{(S+1)-1}}{(S+1) - 1} dv_{N+1} \\
&= (w_{N+1} - 2v_{N+1}) \frac{-(1 - v_{N+1})^{(S+1)}}{(S+1)((S+1) - 1)} \Big|_0^1 - \int_0^1 \frac{2(1 - v_{N+1})^{(S+1)}}{(S+1)((S+1) - 1)} dv_{N+1} \\
&= \frac{w_{N+1}}{(S+1)((S+1) - 1)} + \frac{2(1 - v_{N+1})^{(S+1)+1}}{(S+1)((S+1) - 1)((S+1) + 1)} \Big|_0^1.
\end{aligned}$$

■

Propositions 3.4.6 and 3.4.8(RFA Expected Buyer Surplus and Bid Strategy) The expected surplus to a buyer participating in the N^{th} -to-last period of a PDSM alternating auction sequence is

$$D_N(w_N) = \frac{w_N^B}{B} + \frac{N-1}{2B(B+1)} + \sum_{p=0}^{\frac{N-3}{2}} \frac{\frac{1}{2} - \frac{2}{S-2p}}{B}$$

when N is odd and

$$D_N(w_N) = \frac{w_N - \frac{2}{S+1}}{B} + \frac{N}{2B(B+1)} + \sum_{p=1}^{\frac{N-2}{2}} \frac{\frac{1}{2} - \frac{2}{S-2p+1}}{B}$$

when N is even.

For odd N (i.e., when the buyer is a bidder), the optimal bid strategy is $b_N(w_N) = w_N$.

Proof of Propositions 3.4.6 and 3.4.8

We first assume that there is only one auction in the sequence. Since it is a forward auction, the expected surplus is the same as when $N = 1$ in the sequence of forward auctions, i.e., $D_1 = \frac{(w_1)^B}{B}$.

The expected surplus when $N = 2$ comes from initiating the auction in the first period and bidding in the second period. The expected surplus is given by

$$\begin{aligned}
D_2 &= \frac{1}{B} \int_0^1 \left((w_2 - v_{(2)}) S(S-1) (1 - F(v_{(2)}))^{S-2} F(v_{(2)}) f(v_{(2)}) \right) dv_{(2)} \\
&\quad + \int_0^1 \int_0^{w_1} (w_1 - w_{(2)}) (B-1) F(w_{(2)})^{B-2} f(w_{(2)}) dw_{(2)} f w_1 dw_1 \\
&= \frac{1}{B} \int_0^1 \left((w_2 - v_{(2)}) S(S-1) (1 - v_{(2)})^{S-2} v_{(2)} \right) dv_{(2)} \\
&\quad + \int_0^1 \int_0^{w_1} (w_1 - w_{(2)}) (B-1) w_{(2)}^{B-2} dw_{(2)} dw_1 \\
&= \frac{S(S-1)}{B} \int_0^1 \left((w_2 v_{(2)} - v_{(2)}^2) (1 - v_{(2)})^{S-2} \right) dv_{(2)} \\
&\quad + (B-1) \int_0^1 \int_0^{w_1} (w_1 w_{(2)}^{B-2} - w_{(2)}^{B-1}) dw_{(2)} dw_1 \\
&= \frac{S(S-1)}{B} \int_0^1 \left(w_2 - 2v_{(2)} \frac{(1 - v_{(2)})^{S-1}}{S-1} \right) dv_{(2)} + (B-1) \int_0^1 \left(\frac{w_1 v_{(2)}^{B-1}}{B-1} - \frac{v_{(2)}^B}{B} \right) \Big|_0^{w_1} dw_1 \quad (22) \\
&= \frac{S(S-1)}{B} \left[\frac{w_2}{S(S-1)} + \frac{2(1 - v_{(2)})^{S+1}}{S(S-1)(S+1)} \right] \Big|_0^1 + (B-1) \int_0^1 \left(\frac{w_1^B}{B-1} - \frac{w_1^B}{B} \right) dw_1 \quad (23) \\
&= \frac{S(S-1)}{B} \left[\frac{w_2}{S(S-1)} - \frac{2}{S(S-1)(S+1)} \right] + (B-1) \left(\frac{w_1^{B+1}}{(B-1)(B+1)} - \frac{w_1^{B+1}}{B(B+1)} \right) \Big|_0^1 \\
&= \frac{w_2 - \frac{2}{S+1}}{B} + (B-1) \left(\frac{1}{(B-1)(B+1)} - \frac{1}{B(B+1)} \right) \\
&= \frac{w_2 - \frac{2}{S+1}}{B} + (B-1) \left(\frac{1}{B(B-1)(B+1)} \right) \\
&= \frac{w_2 - \frac{2}{S+1}}{B} + \frac{1}{B(B+1)}
\end{aligned}$$

where (26) follows from the integration-by-parts of the preceding line as follows:

$$\begin{aligned}
&\int_0^1 (w_2 v_{(2)} - v_{(2)}^2) (1 - v_{(2)})^{S-2} dv_{(2)} \\
&= (w_2 v_{(2)} - v_{(2)}^2) \frac{(-(1 - v_{(2)})^{S-1})}{S-1} \Big|_0^1 + \int_0^1 (w_2 - 2v_{(2)}) \frac{(1 - v_{(2)})^{S-1}}{S-1} dv_{(2)} \\
&= \int_0^1 (w_2 - 2v_{(2)}) \frac{(1 - v_{(2)})^{S-1}}{S-1} dv_{(2)},
\end{aligned}$$

and the term (27) follows from the Integration by Parts of the preceding line as follows:

$$\begin{aligned}
&\int_0^1 (w_2 - 2v_{(2)}) \frac{(1 - v_{(2)})^{S-1}}{S-1} dv_{(2)} \\
&= (w_2 - 2v_{(2)}) \frac{(-(1 - v_{(2)})^S)}{S(S-1)} \Big|_0^1 - \int_0^1 \frac{2(1 - v_{(2)})^S}{S(S-1)} dv_{(2)} \\
&= \frac{w_2}{S(S-1)} + \frac{2(1 - v_{(2)})^{S+1}}{S(S-1)(S+1)} \Big|_0^1.
\end{aligned}$$

Note that because these are sequential English auctions, for $N = 1$ the symmetric equilibrium strategy is to bid $b_1(w_1) = w_1$ (Milgrom and Weber, 2000).

For the remaining sequences, we will assume that N is odd and prove by induction the result when $N + 1$ is even. Then, we assume that N is even and prove the result when $N + 1$ is odd.

Assume the expected surplus to a buyer participating in the N^{th} -to-last period of a PDSM alternating auction that begins with a forward auction (i.e., N is odd) is

$$D_N = \frac{w_N^B}{B} + \frac{N-1}{2B(B+1)} + \sum_{p=0}^{\frac{N-3}{2}} \frac{\frac{1}{2} - \frac{2}{S-2p}}{B}.$$

The total expected surplus in an $N + 1$ period sequence of alternating auctions is

$$\begin{aligned} D_{N+1} &= \frac{1}{B} \int_0^1 (w_{N+1} - v_{(2)})(S+1)((S+1)-1)(1-F(v_{(2)})^{(S+1)-2})F(v_{(2)})f(v_{(2)})dv_{(2)} \\ &\quad + \int_0^1 D_N f(w_N)dw_N \\ &= \frac{1}{B} \int_0^1 (w_{N+1} - v_{(2)})(S+1)((S+1)-1)(1-v_{(2)}^{(S+1)-2})v_{(2)}dv_{(2)} \\ &\quad + \int_0^1 \left(\frac{w_N^B}{B} + \frac{N-1}{2B(B+1)} + \sum_{p=0}^{\frac{N-3}{2}} \frac{\frac{1}{2} - \frac{2}{S-2p}}{B} \right) dw_N \\ &= \frac{(S+1)((S+1)-1)}{B} \int_0^1 (w_{N+1}v_{(2)} - v_{(2)}^2)(1-v_{(2)})^{(S+1)-2}dv_{(2)} \\ &\quad + \left(\frac{w_N^{B+1}}{B(B+1)} + \frac{N-1}{2B(B+1)} + \sum_{p=0}^{\frac{N-3}{2}} \frac{\frac{1}{2} - \frac{2}{S-2p}}{B} \right) \Big|_0^1 \\ &= \frac{1}{B} \left(w_{N+1} - \frac{2}{(S+1)+1} \right) + \frac{1}{B(B+1)} + \frac{N-1}{2B(B+1)} + \sum_{p=0}^{\frac{N-3}{2}} \frac{\frac{1}{2} - \frac{2}{(S+1)-2p-1}}{B} \quad (24) \\ &= \frac{w_{N+1} - \frac{2}{(S+1)+1}}{B} + \frac{N+1}{2B(B+1)} + \sum_{p=1}^{\frac{(N+1)-2}{2}} \frac{\frac{1}{2} - \frac{2}{(S+1)-2p+1}}{B}, \end{aligned}$$

where (28) follows from the integration-by-parts of the preceding expression as follows:

$$\begin{aligned}
& \int_0^1 (w_{N+1}v_{(2)} - v_{(2)}^2)(1 - v_{(2)})^{S-2} dv_{(2)} \\
&= (w_{N+1}v_{(2)} - v_{(2)}^2) \frac{-(1 - v_{(2)})^{S-1}}{S-1} \Big|_0^1 + \int_0^1 (w_{N+1} - 2v_{(2)}) \frac{(1 - v_{(2)})^{S-1}}{S-1} dv_{(2)} \\
&= \int_0^1 (w_{N+1} - 2v_{(2)}) \frac{(1 - v_{(2)})^{S-1}}{S-1} dv_{(2)} \\
&= (w_{N+1} - 2v_{(2)}) \frac{-(1 - v_{(2)})^S}{S(S-1)} \Big|_0^1 - \int_0^1 \frac{2(1 - v_{(2)})^S}{S(S-1)} dv_{(2)} \\
&= \frac{w_{N+1}}{S(S-1)} + \frac{2(1 - v_{(2)})^{S+1}}{S(S-1)(S+1)} \Big|_0^1.
\end{aligned}$$

When N is an even number the result holds.

Now assume that the expected surplus to the buyer participating in the N^{th} -to-last period of a PDSM alternating auction sequence that begins with a reverse auction (i.e., N is even) is

$$D_N = \frac{w_N - \frac{2}{S+1}}{B} + \frac{N}{2B(B+1)} + \sum_{p=1}^{\frac{N-2}{2}} \frac{\frac{1}{2} - \frac{2}{S-2p+1}}{B}.$$

Using this assumption, we derive the bid function for a buyer in the N^{th} -to-last period as follows:

$$\begin{aligned}
D_{N+1} &= \int_0^r (w_{N+1} - b(w_{(2)}))(B-1)F(w_{(2)})^{B-2}f(w_{(2)})dw_{(2)} \\
&+ \int_0^1 D_N f(w_N)dw_N.
\end{aligned} \tag{25}$$

We obtain the first order conditions using Leibnitz's rule. Although r is a function of w_{N+1} , for notational simplicity, we write r in the equation instead of writing it as a function of a private valuation, which yields

$$\frac{dD_{N+1}}{dr} = (B-1)(w_{N+1} - b_{N+1}(r))(r)^{B-2} \equiv 0.$$

We solve for $b_{N+1}(r)$, which is

$$b_{N+1}(r) = w_{N+1}.$$

The right hand side of the bid function does not depend on r , so

$$b_{N+1}(w_{N+1}) = w_{N+1},$$

which is positive. The derivative of the expected profit, $D'_{N+1} > (<) 0$ when $b_{N+1}(w_{N+1}) < (>) w_{N+1}$. Therefore, $b_{N+1}(w_{N+1})$ is the optimal equilibrium response for the bidder of type w_{N+1} in the first of $N + 1$ auctions.

We substitute this optimal bid function into equation (29). Because the strategic buyer will bid as his true type w_{N+1} , we replace r with w_{N+1} in the limits of integration. We obtain

$$\begin{aligned}
D_{N+1} &= \int_0^{w_{N+1}} (w_{N+1} - w_{(2)})(B-1)F(w_{(2)})^{B-2}f(w_{(2)})dw_{(2)} \\
&\quad + \int_0^1 D_N f(w_N)dw_N \\
&= \int_0^{w_{N+1}} (w_{N+1} - w_{(2)})(B-1)w_{(2)}^{B-2}dw_{(2)} \\
&\quad + \int_0^1 \left(\frac{w_N - \frac{2}{S+1}}{B} + \frac{N}{2B(B+1)} + \sum_{p=1}^{\frac{N-2}{2}} \frac{\frac{1}{2} - \frac{2}{S-2p+1}}{B} \right) dw_N \\
&= (B-1) \int_0^{w_{N+1}} (w_{N+1}w_{(2)}^{B-2} - w_{(2)}^{B-1})dw_{(2)} \\
&\quad + \int_0^1 \left(\frac{w_N - \frac{2}{S+1}}{B} + \frac{N}{2B(B+1)} + \sum_{p=1}^{\frac{N-2}{2}} \frac{\frac{1}{2} - \frac{2}{S-2p+1}}{B} \right) dw_N \\
&= (B-1) \left[\frac{w_{N+1}w_{(2)}^{B-1}}{B-1} - \frac{w_{(2)}^B}{B} \right]_0^{w_{N+1}} \\
&\quad + \frac{\frac{1}{2} - \frac{2}{S+1}}{B} + \frac{N}{2B(B+1)} + \sum_{p=1}^{\frac{N-2}{2}} \frac{\frac{1}{2} - \frac{2}{S-2p+1}}{B} \\
&= (B-1) \left(\frac{w_{N+1}^B}{B-1} - \frac{w_{N+1}^B}{B} \right) \\
&\quad + \frac{\frac{1}{2} - \frac{2}{S+1}}{B} + \frac{N}{2B(B+1)} + \sum_{p=1}^{\frac{N-2}{2}} \frac{\frac{1}{2} - \frac{2}{S-2p+1}}{B} \\
&= \frac{w_{N+1}^B}{B} + \frac{\frac{1}{2} - \frac{2}{S+1}}{B} + \frac{N}{2B(B+1)} + \sum_{p=1}^{\frac{N-2}{2}} \frac{\frac{1}{2} - \frac{2}{(S+1)-2p}}{B} \\
&= \frac{w_{N+1}^B}{B} + \frac{N}{2B(B+1)} + \frac{\frac{1}{2} - \frac{2}{(S+1)}}{B} + \frac{\frac{1}{2} - \frac{2}{(S+1)-2}}{B} \\
&\quad + \frac{\frac{1}{2} - \frac{2}{(S+1)-4}}{B} + \dots + \frac{\frac{1}{2} - \frac{2}{(S+1)-N+2}}{B} \\
&= \frac{w_{N+1}^B}{B} + \frac{(N+1)-1}{2B(B+1)} + \sum_{p=0}^{\frac{(N+1)-3}{2}} \frac{\frac{1}{2} - \frac{2}{(S+1)-2p}}{B}.
\end{aligned}$$

Hence for when N is an odd number, the result holds. ■

Proposition 3.4.7(FA Expected Buyer Surplus and Bid Strategy) The expected surplus to a buyer competing in the N^{th} -to-last period of a PDSM forward auction sequence is

$$D_N = \frac{(w_N)^B}{B} + \frac{N-1}{B(B+1)},$$

and the optimal equilibrium bid strategy is

$$b_N(w_N) = w_N.$$

Proof of Proposition 3.4.7

We first consider the last auction in the sequence (i.e., $N = 1$). Because this auction is an English auction, the symmetric equilibrium bid strategy is to bid $b_1(w_1) = w_1$ (Milgrom and Weber, 2000), where w_1 is the bidder's valuation in this period. The expected surplus to the buyer is the difference between his valuation and that of the second highest buyer multiplied by the probability of that buyer actually having the second highest valuation of all other buyers. This is given by

$$\begin{aligned} D_1 &= \int_0^{w_1} (w_1 - w_{(2)})(B-1)F(w_{(2)})^{B-2}f(w_{(2)})dw_{(2)} \\ &= (B-1) \int_0^{w_1} (w_1 - w_{(2)})w_{(2)}^{B-2}dw_{(2)} \\ &= (B-1) \int_0^{w_1} (w_1 w_{(2)}^{B-2} - w_{(2)}^{B-1})dw_{(2)} \\ &= (B-1) \left[\frac{w_1 w_{(2)}^{B-1}}{B-1} - \frac{w_{(2)}^B}{B} \right] \Big|_0^{w_1} \\ &= (B-1) \left[\frac{w_1^B}{B-1} - \frac{w_1^B}{B} \right] \\ &= (B-1) \left[\frac{Bw_1^B - Bw_1^B + w_1^B}{B(B-1)} \right] \\ &= \frac{w_1^B}{B}. \end{aligned}$$

Assume the total expected surplus to the buyer competing in the N^{th} -to-last period of a PDSM forward auction is

$$D_N = \frac{w_N^B}{B} + \frac{N-1}{B(B+1)},$$

and the optimal bid strategy is

$$b_N(w_N) = w_N.$$

The expected surplus to the buyer of winning in the first of $N + 1$ periods plus the expected surplus of winning in future periods when the future private valuations are unknown is

$$\begin{aligned} D_{N+1} = & \int_0^r (w_{N+1} - b_{N+1}(w_{(2)}))(B-1)F(w_{(2)})^{B-2}f(w_{(2)})dw_{(2)} \\ & + \int_0^1 \frac{w_N^B}{B} f(w_N)dw_N + \frac{N-1}{B(B+1)}, \end{aligned} \quad (26)$$

where r is the bidder type of the buyer.

We obtain the first order conditions using Leibnitz's rule. Although r is a function of w_{N+1} , for notational simplicity, we write r in the equation instead of writing it as a function of a private valuation, which yields

$$\frac{dD_{N+1}}{dr} = (B-1)(w_{N+1} - b_{N+1}(r))(r)^{B-2} \equiv 0.$$

We now solve for $b_{N+1}(r)$, which is

$$b_{N+1}(r) = w_{N+1}.$$

The right hand side of the bid function does not depend on r , so

$$b_{N+1}(w_{N+1}) = w_{N+1}.$$

This function is nonnegative. The derivative of the expected profit, $D'_{N+1}(w_{N+1}) > (<) 0$ when $b_{N+1}(w_{N+1}) < (>) w_{N+1}$. Therefore $b_{N+1}(w_{N+1})$ is the optimal equilibrium response for the bidder of type w_{N+1} in the $(N+1)^{st}$ -to-last auction in the sequence.

The above equation states that the buyer should bid as his true type.

We substitute this optimal bid function into (30) as the bid of the competitive buyer who has type $w_{(2)}$. Because a strategic buyer will bid as his true type, w_{N+1} , we replace r with

w_{N+1} in the limits of integration. We can now simplify D_{N+1} as follows:

$$\begin{aligned}
D_{N+1} &= \int_0^{w_{N+1}} (w_{N+1} - b_{N+1}(w_{(2)}))(B-1)F(w_{(2)})^{B-2}f(w_{(2)})dw_{(2)} \\
&\quad + \int_0^1 D_N(w_N)f(w_N)dw_N \\
&= (B-1) \int_0^{w_{N+1}} (w_{N+1} - w_{(2)})w_{(2)}^{B-2}dw_{(2)} \\
&\quad + \int_0^1 \frac{w_N^B}{B} + \frac{N-1}{B(B+1)}dw_N \\
&= (B-1) \int_0^{w_{N+1}} (w_{N+1}w_{(2)}^{B-2} - w_{(2)}^{B-1})dw_{(2)} \\
&\quad + \frac{1}{B(B+1)} + \frac{N-1}{B(B+1)} \\
&= (B-1) \left[\frac{w_{N+1}w_{(2)}^{B-1}}{B-1} - \frac{w_{(2)}^B}{B} \right] \Big|_0^{w_1} \\
&\quad + \frac{N}{B(B+1)} \\
&= (B-1) \left[\frac{w_{N+1}^B}{B-1} - \frac{w_{N+1}^B}{B} \right] + \frac{N}{B(B+1)} \\
&= (B-1) \left[\frac{Bw_{N+1}^B - Bw_{N+1}^B + w_{N+1}^B}{B(B-1)} \right] + \frac{N}{B(B+1)} \\
&= \frac{w_{N+1}^B}{B} + \frac{N}{B(B+1)},
\end{aligned}$$

where the first term, $\frac{w_{N+1}^B}{B}$, is the expected surplus from winning the $(N+1)^{st}$ -to-last auction and the second term is the total future expected surplus. ■

APPENDIX B

PROOFS OF PROPOSITIONS 3.5.1-3.5.6

In this appendix, we derive the inequalities in Table 1. These inequalities correspond to the buyer and supplier preference of each auction design. Each inequality is simplified to give the number of buyers needed to make the preference of an auction design hold.

The expected buyer surplus functions integrated with respect to the private valuations are as follows:

Reverse Auction

$$\begin{aligned}
 D_N^{RA} &= \int_0^1 \left(\frac{w_N - \frac{2}{S+1}}{B} + \sum_{p=1}^{N-1} \frac{\frac{1}{2} - \frac{2}{S-p+1}}{B} \right) dw_N \\
 &= \frac{\frac{1}{2} - \frac{2}{S+1}}{B} + \sum_{p=1}^{N-1} \frac{\frac{1}{2} - \frac{2}{S-p+1}}{B} \\
 &= \sum_{p=0}^{N-1} \frac{\frac{1}{2} - \frac{2}{S-p+1}}{B}.
 \end{aligned}$$

Forward Auction

$$\begin{aligned}
 D_N^{FA} &= \int_0^1 \left(\frac{w_N^B}{B} + \frac{N-1}{B(B+1)} \right) dw_N \\
 &= \frac{1}{B(B+1)} + \frac{N-1}{B(B+1)} \\
 &= \frac{N}{B(B+1)}.
 \end{aligned}$$

Alternating Auction

When N is even,

$$\begin{aligned}
 D_N^{RFA} &= \int_0^1 \left(\frac{w_N - \frac{2}{S+1}}{B} + \frac{N}{2B(B+1)} + \sum_{p=1}^{\frac{N-2}{2}} \frac{\frac{1}{2} - \frac{2}{S-2p+1}}{B} \right) dw_N \\
 &= \frac{\frac{1}{2} - \frac{2}{S+1}}{B} + \frac{N}{2B(B+1)} + \sum_{p=1}^{\frac{N-2}{2}} \frac{\frac{1}{2} - \frac{2}{S-2p+1}}{B} \\
 &= \frac{N}{2B(B+1)} + \sum_{p=0}^{\frac{N-2}{2}} \frac{\frac{1}{2} - \frac{2}{S-2p+1}}{B},
 \end{aligned}$$

and when N is odd,

$$\begin{aligned}
D_N^{RFA} &= \int_0^1 \left(\frac{w^B}{B} + \frac{N-1}{2B(B+1)} + \sum_{p=0}^{\frac{N-3}{2}} \frac{\frac{1}{2} - \frac{2}{S-2p}}{B} \right) dw_N \\
&= \frac{1}{B(B+1)} + \frac{N-1}{2B(B+1)} + \sum_{p=0}^{\frac{N-3}{2}} \frac{\frac{1}{2} - \frac{2}{S-2p}}{B} \\
&= \frac{N+1}{2B(B+1)} + \sum_{p=0}^{\frac{N-3}{2}} \frac{\frac{1}{2} - \frac{2}{S-2p}}{B}.
\end{aligned}$$

Proposition 3.5.1 (Buyer Reverse Auction vs. Forward Auction) A surplus-maximizing buyer will be indifferent between a reverse auction and a forward auction when $B = \frac{N}{\lambda} - 1$, where $\lambda = \sum_{p=0}^{N-1} (\frac{1}{2} - \frac{2}{S-p+1})$.

Under the standard assumptions that $S \geq N+1$ and $N \geq 2$, when $S \geq 5$ or both $S \geq 4$ and $N = 2$, the buyer will prefer the reverse auction if and only if $B > \frac{N}{\lambda} - 1$. When $S = N+1$ for $N \in \{2, 3\}$, the buyer will never prefer the reverse auction (because no B exists for which $B < \frac{N}{\lambda} - 1$).

Proof of Proposition 3.5.1

Because the buyer looks to maximize expected surplus, he will be indifferent whenever $D_N^{RA} = D_N^{FA}$. As shown above, this is equivalent to

$$\begin{aligned}
\sum_{p=0}^{N-1} \frac{\frac{1}{2} - \frac{2}{S-p+1}}{B} &= \frac{N}{B(B+1)}, \\
\frac{\lambda}{B} &= \frac{N}{B(B+1)}, \\
B+1 &= \frac{N}{\lambda}, \\
B &= \frac{N}{\lambda} - 1.
\end{aligned}$$

By the same reasoning, the buyer will prefer a reverse auction if and only if $D_N^{RA} > D_N^{FA}$, i.e., if and only if

$$\begin{aligned}
\sum_{p=0}^{N-1} \frac{\frac{1}{2} - \frac{2}{S-p+1}}{B} &> \frac{N}{B(B+1)}, \\
\frac{\lambda}{B} &> \frac{N}{B(B+1)}, \\
\lambda &> \frac{N}{B+1}.
\end{aligned}$$

Because we divide by λ to simplify this expression with respect to B , the key now is the sign of λ . If $\lambda > 0$, we can continue as above and conclude that the buyer will prefer a reverse auction if and only if $B > \frac{N}{\lambda} - 1$. However, if $\lambda < 0$, the sign of the inequality will switch when we divide both sides by λ , and we conclude that the buyer will prefer a reverse auction if and only if $B < \frac{N}{\lambda} - 1$.

To determine the sign of λ , we use the standard assumptions that $S \geq N + 1$ and $N \geq 2$ (so that competition exists in every auction).

Case 1: $S \geq 5$. In this case,

$$\begin{aligned}\lambda &= \sum_{p=0}^{N-1} \left(\frac{1}{2} - \frac{2}{S-p+1} \right) \\ &= \left(\frac{1}{2} - \frac{2}{S-(N-1)+1} \right) + \left(\frac{1}{2} - \frac{2}{S-(N-2)+1} \right) + \left(\frac{1}{2} - \frac{2}{S-(N-3)+1} \right) \\ &\quad + \left(\frac{1}{2} - \frac{2}{S-(N-4)+1} \right) + \sum_{p=0}^{N-5} \left(\frac{1}{2} - \frac{2}{S-p+1} \right).\end{aligned}$$

The terms in each summation decrease as the index increases since $\frac{1}{2} - \frac{2}{S+1} > \frac{1}{2} - \frac{2}{S} > \frac{1}{2} - \frac{2}{S-2} > \dots > \frac{1}{2} - \frac{2}{S-N+2}$. So the lowest value in the summation will be $\frac{1}{2} - \frac{2}{S-N+2}$. In addition, the lowest value of S will yield the lowest value for each difference (i.e., $\frac{1}{2} - \frac{2}{S-p+1}$). Since $S \geq N + 1$, we substitute $S = N + 1$ for S to obtain

$$\lambda \geq \left(\frac{1}{2} - \frac{2}{3} \right) + \left(\frac{1}{2} - \frac{2}{4} \right) + \left(\frac{1}{2} - \frac{2}{5} \right) + \left(\frac{1}{2} - \frac{2}{6} \right) + \sum_{p=0}^{N-5} \left(\frac{1}{2} - \frac{2}{7} \right).$$

Each of the terms in the summation is positive (since $\frac{1}{2} > \frac{2}{7}$) and the four terms outside the summation are $4\frac{1}{2} - \frac{2}{6} - \frac{2}{5} - \frac{2}{4} - \frac{2}{3} = 2 - \frac{114}{60} > 0$; therefore, $\lambda > 0$ and the buyer will prefer a reverse auction if and only if $B > \frac{N}{\lambda} - 1$.

Case 2: $S = 4$ and $N = 2$. If $N = 2$ then $\lambda = \left(\frac{1}{2} - \frac{2}{5} \right) + \left(\frac{1}{2} - \frac{2}{4} \right) > 0$. Therefore, in this case the buyer will prefer a reverse auction whenever $B > \frac{N}{\lambda} - 1$.

Case 3: $S = N + 1$ and $N \in \{2, 3\}$. This case has two subcases. If $S = 3$ and $N = 2$, then $\lambda = \left(\frac{1}{2} - \frac{2}{4} \right) + \left(\frac{1}{2} - \frac{2}{3} \right) < 0$. If $S = 4$ and $N = 3$, then $\lambda = \left(\frac{1}{2} - \frac{2}{5} \right) + \left(\frac{1}{2} - \frac{2}{4} \right) + \left(\frac{1}{2} - \frac{2}{3} \right) < 0$. Therefore, in this case the buyer will prefer a reverse auction if and only if $B < \frac{N}{\lambda} - 1$. However, it is easy to show that in fact the buyer will never prefer a reverse auction in this case. Since $\lambda < 0$, $\frac{N}{\lambda} - 1 < 0$ and it is impossible for B to be less than 1 in any auction. ■

Proposition 3.5.2 (Buyer Reverse Auction vs. Alternating Auction) A surplus-maximizing buyer will be indifferent between a sequence of reverse auctions and a sequence of alternating auctions when $B = \frac{N}{2(\lambda-\rho)} - 1$, where

$$\lambda = \sum_{p=0}^{N-1} \frac{1}{2} - \frac{2}{S-p+1}$$

and

$$\rho = \begin{cases} \sum_{p=0}^{\frac{N-2}{2}} \frac{1}{2} - \frac{2}{S-2p+1}, & \text{when } N \text{ is even} \\ \sum_{p=0}^{\frac{N-3}{2}} \frac{1}{2} - \frac{2}{S-2p}, & \text{when } N \text{ is odd.} \end{cases}$$

Under the standard assumptions that $S \geq N + 1$ and $N \geq 2$, when $S \geq 6$ or when both $S = 5$ and $N = 2$ or $N = 3$, the buyer will prefer the reverse auction if and only if $B > \frac{N}{2(\lambda-\rho)} - 1$. When $S = N + 1$ for $N \in \{2, 3, 4\}$, the buyer will never prefer the reverse auction (because no B exists for which $B < \frac{N}{2(\lambda-\rho)} - 1$). When $S = 4$ and $N = 2$ (the only remaining case), the buyer again will never prefer the reverse auction.

Proof of Proposition 3.5.2

Because the buyer looks to maximize expected surplus, he will be indifferent whenever $D_N^{RA} = D_N^{RFA}$. As shown above, this is equivalent to

$$\begin{aligned} D_N^{RA} &= D_N^{RFA} \\ \frac{\lambda}{B} &= \frac{N}{2B(B+1)} + \frac{\rho}{B} \\ \lambda - \rho &= \frac{N}{2(B+1)} \\ (B+1)(\lambda - \rho) &= \frac{N}{2} \\ B(\lambda - \rho) &= \frac{N}{2} - (\lambda - \rho) \\ B &= \frac{\frac{N}{2} - (\lambda - \rho)}{\lambda - \rho} \\ B &= \frac{N}{2(\lambda - \rho)} - 1, \end{aligned}$$

where

$$\lambda = \sum_{p=0}^{N-1} \frac{1}{2} - \frac{2}{S-p+1}$$

and

$$\rho = \begin{cases} \sum_{p=0}^{\frac{N-2}{2}} \frac{1}{2} - \frac{2}{S-2p+1}, & \text{when } N \text{ is even} \\ \sum_{p=0}^{\frac{N-3}{2}} \frac{1}{2} - \frac{2}{S-2p}, & \text{when } N \text{ is odd.} \end{cases}$$

By the same reasoning, the buyer will prefer a reverse auction if and only if $D_N^{RA} > D_N^{RFA}$, i.e., if and only if

$$\begin{aligned} \frac{\lambda}{B} &> \frac{N}{2B(B+1)} + \frac{\rho}{B} \\ \lambda - \rho &> \frac{N}{2(B+1)} \\ (B+1)(\lambda - \rho) &> \frac{N}{2} \\ B(\lambda - \rho) &> \frac{N}{2} - (\lambda - \rho) \end{aligned}$$

Because we divide by $\lambda - \rho$ to simplify this expression with respect to B , the key now is the sign of $(\lambda - \rho)$. If $(\lambda - \rho) > 0$, we can continue as above and conclude that the buyer will prefer a reverse auction if and only if $B > \frac{N}{2(\lambda - \rho)} - 1$. However, if $(\lambda - \rho) < 0$, the sign of the inequality will switch when we divide both sides by $(\lambda - \rho)$, and we conclude that the buyer will prefer a reverse auction if and only if $B < \frac{N}{2(\lambda - \rho)} - 1$.

To determine the sign of $(\lambda - \rho)$, we use the standard assumptions that $S \geq N + 1$ and $N \geq 2$ (so that competition exists in every auction). We begin with the case that N is even.

Case 1: $S \geq 6$. In this case,

$$\begin{aligned} \lambda &= \sum_{p=0}^{N-1} \frac{1}{2} - \frac{2}{S-p+1} \\ &= \left(\frac{1}{2} - \frac{2}{S-(N-1)+1}\right) + \left(\frac{1}{2} - \frac{2}{S-(N-2)+1}\right) + \left(\frac{1}{2} - \frac{2}{S-(N-3)+1}\right) \\ &\quad + \left(\frac{1}{2} - \frac{2}{S-(N-4)+1}\right) + \left(\frac{1}{2} - \frac{2}{S-(N-5)+1}\right) + \sum_{p=0}^{N-6} \left(\frac{1}{2} - \frac{2}{S-p+1}\right). \end{aligned}$$

and

$$\begin{aligned} \rho &= \sum_{p=0}^{\frac{N-2}{2}} \frac{1}{2} - \frac{2}{S-2p+1} \text{ when } N \text{ is even} \\ &= \left(\frac{1}{2} - \frac{2}{S-(N-2)+1}\right) + \left(\frac{1}{2} - \frac{2}{S-(N-4)+1}\right) + \sum_{p=0}^{\frac{N-6}{2}} \left(\frac{1}{2} - \frac{2}{S-2p+1}\right). \end{aligned}$$

This makes $(\lambda - \rho)$ equivalent to

$$\begin{aligned}\lambda - \rho &= \left(\frac{1}{2} - \frac{2}{S - (N-1) + 1}\right) + \left(\frac{1}{2} - \frac{2}{S - (N-3) + 1}\right) \\ &+ \left(\frac{1}{2} - \frac{2}{S - (N-5) + 1}\right) + \sum_{p=0}^{N-6} \left(\frac{1}{2} - \frac{2}{S - p + 1}\right) - \sum_{p=0}^{\frac{N-6}{2}} \left(\frac{1}{2} - \frac{2}{S - 2p + 1}\right).\end{aligned}$$

The p^{th} term in the second summation is exactly equal to the $(2p)^{th}$ term in the first summation. Therefore, $\lambda - \rho$ simplifies to

$$\begin{aligned}\lambda - \rho &= \left(\frac{1}{2} - \frac{2}{S - (N-1) + 1}\right) + \left(\frac{1}{2} - \frac{2}{S - (N-3) + 1}\right) \\ &+ \left(\frac{1}{2} - \frac{2}{S - (N-5) + 1}\right) + \sum_{p=0}^{\frac{N-6}{2}} \left(\frac{1}{2} - \frac{2}{S - 2p}\right).\end{aligned}$$

The terms in each summation decrease as the index increases since $\frac{1}{2} - \frac{2}{S} > \frac{1}{2} - \frac{2}{S-2} > \frac{1}{2} - \frac{2}{S-4} > \dots > \frac{1}{2} - \frac{2}{S-N+6}$. So the lowest value in the summation will be $\frac{1}{2} - \frac{2}{S-N+6}$. In addition, the lowest value of S will yield the lowest value for each difference (i.e., $\frac{1}{2} - \frac{2}{S-2p}$). Since $S \geq N + 1$,

$$(\lambda - \rho) \geq \left(\frac{1}{2} - \frac{2}{3}\right) + \left(\frac{1}{2} - \frac{2}{5}\right) + \left(\frac{1}{2} - \frac{2}{7}\right) + \sum_{p=0}^{\frac{N-6}{2}} \left(\frac{1}{2} - \frac{2}{9}\right).$$

Each of the terms in the summation is positive (since $\frac{1}{2} > \frac{2}{9}$) and the three terms outside the summation are $\frac{3}{2} - \frac{2}{3} - \frac{2}{5} - \frac{2}{7} = \frac{3}{2} - \frac{142}{105} > 0$; therefore, $\lambda - \rho > 0$ and the buyer will prefer a reverse auction if and only if $B > \frac{N}{2(\lambda - \rho)} - 1$.

We continue in this case and consider the sign of $\lambda - \rho$ when N is odd. When N is odd,

$$\begin{aligned}\rho &= \sum_{p=0}^{\frac{N-3}{2}} \frac{1}{2} - \frac{2}{S - 2p} \\ &= \left(\frac{1}{2} - \frac{2}{S - (N-3)}\right) + \left(\frac{1}{2} - \frac{2}{S - (N-5)}\right) + \sum_{p=0}^{\frac{N-7}{2}} \left(\frac{1}{2} - \frac{2}{S - 2p}\right).\end{aligned}$$

This makes $(\lambda - \rho)$ equivalent to

$$\begin{aligned}\lambda - \rho &= \left(\frac{1}{2} - \frac{2}{S - (N-1) + 1}\right) + \left(\frac{1}{2} - \frac{2}{S - (N-3) + 1}\right) \\ &+ \left(\frac{1}{2} - \frac{2}{S - (N-5) + 1}\right) + \sum_{p=0}^{N-6} \left(\frac{1}{2} - \frac{2}{S - p + 1}\right) - \sum_{p=0}^{\frac{N-7}{2}} \left(\frac{1}{2} - \frac{2}{S - 2p}\right).\end{aligned}$$

The terms of the first summation equal those of the second summation when p in the first summation is odd which means the first summation has twice as many terms as the second summation. Therefore, $\lambda - \rho$ simplifies to

$$\begin{aligned}\lambda - \rho &= \left(\frac{1}{2} - \frac{2}{S - (N-1) + 1}\right) + \left(\frac{1}{2} - \frac{2}{S - (N-3) + 1}\right) \\ &+ \left(\frac{1}{2} - \frac{2}{S - (N-5) + 1}\right) + \sum_{p=0}^{\frac{N-6}{2}} \left(\frac{1}{2} - \frac{2}{S - 2p}\right).\end{aligned}$$

The terms in each summation decrease as the index increases since $\frac{1}{2} - \frac{2}{S} > \frac{1}{2} - \frac{2}{S-2} > \frac{1}{2} - \frac{2}{S-4} > \dots$. In addition, the lowest value of S will yield the lowest value for each difference (i.e., $\frac{1}{2} - \frac{2}{S-2p}$). Since $S \geq N+1$, we have

$$(\lambda - \rho) \geq \left(\frac{1}{2} - \frac{2}{3}\right) + \left(\frac{1}{2} - \frac{2}{5}\right) + \left(\frac{1}{2} - \frac{2}{7}\right) + \sum_{p=0}^{\frac{N-6}{2}} \left(\frac{1}{2} - \frac{2}{9}\right).$$

Each of the terms in the summation is positive (since $\frac{1}{2} > \frac{2}{9}$) and the three terms outside the summation are $\frac{3}{2} - \frac{2}{3} - \frac{2}{5} - \frac{2}{7} = \frac{3}{2} - \frac{142}{105} > 0$; therefore, $\lambda - \rho > 0$ and the buyer will prefer a reverse auction if and only if $B > \frac{N}{2(\lambda-\rho)} - 1$.

Case 2: $S = 5$ and $N \in \{2, 3\}$. This case has two subcases. If $N = 2$ then $\lambda - \rho = \frac{1}{2} - \frac{2}{5} > 0$. If $N = 3$ then $\lambda - \rho = \left(\frac{1}{2} - \frac{2}{6}\right) + \left(\frac{1}{2} - \frac{2}{4}\right) > 0$. In this case the buyer will prefer a reverse auction whenever $B > \frac{N}{2(\lambda-\rho)} - 1$.

Case 3: $S = N+1$ and $N \in \{2, 3, 4\}$. This case has three subcases. If $S = 3$ and $N = 2$, then $\lambda - \rho = \frac{1}{2} - \frac{2}{3} < 0$. If $S = 4$ and $N = 3$, then $\lambda - \rho = \left(\frac{1}{2} - \frac{2}{5}\right) + \left(\frac{1}{2} - \frac{2}{3}\right) < 0$. If $S = 5$ and $N = 4$, then $\lambda = \left(\frac{1}{2} - \frac{2}{5}\right) + \left(\frac{1}{2} - \frac{2}{3}\right) < 0$. Therefore, in this case the buyer will prefer a reverse auction if and only if $B < \frac{N}{2(\lambda-\rho)} - 1$. However, it is easy to show that in fact the buyer will never prefer a reverse auction in this case. Since $\lambda - \rho < 0$, $\frac{N}{2(\lambda-\rho)} - 1 < 0$ and it is impossible for B to be less than 1 in any auction.

Case 4: $S = 4$ and $N = 2$. This is the only remaining case for which $S \geq N \geq 2$. In this case, $\lambda - \rho = \frac{1}{2} - \frac{2}{4} = 0$, so we cannot divide by $\lambda - \rho$. Therefore, we return to the

initial inequality, which states that the buyer will prefer the reverse auction if and only if

$$\begin{aligned} D_N^{RA} &> D_N^{RFA} \\ \frac{\lambda}{B} &> \frac{N}{2B(B+1)} + \frac{\rho}{B} \\ \lambda - \rho &> \frac{N}{2(B+1)} \end{aligned}$$

Since both B and N must be positive, it is impossible for this to occur; therefore, the buyer will never prefer a reverse auction in this case. ■

Proposition 3.5.3 (Buyer Forward Auction vs. Alternating Auction) A surplus-maximizing buyer will be indifferent between a sequence of forward auctions and a sequence of alternating auctions when $B = \frac{N}{2\rho} - 1$

where

$$\rho = \begin{cases} \sum_{p=0}^{\frac{N-2}{2}} \frac{1}{2} - \frac{2}{S-2p+1}, & \text{when } N \text{ is even} \\ \sum_{p=0}^{\frac{N-3}{2}} \frac{1}{2} - \frac{2}{S-2p}, & \text{when } N \text{ is odd.} \end{cases}$$

Under the standard assumptions that $S \geq N+1$ and $N \geq 2$, when $S \geq 5$ or when both $S = 4$ and $N = 2$, the buyer will prefer the forward auction if and only if $B < \frac{N}{2\rho} - 1$. When $S = N+1$ for $N \in \{2, 3\}$, the buyer will prefer the forward auction because for any B , $\frac{N}{2(B+1)} > \rho$.

Proof of Proposition 3.5.3

Because the buyer looks to maximize expected surplus, he will be indifferent whenever

$D_N^{FA} = D_N^{RFA}$. As shown above, this is equivalent to

$$\begin{aligned}
D_N^{FA} &= D_N^{RFA} \\
\frac{N}{B(B+1)} &= \frac{1}{B} \left(\frac{N}{2(B+1)} + \rho \right) \\
\frac{N}{B+1} &= \frac{N}{2(B+1)} + \rho \\
\frac{N}{B+1} - \frac{N}{2(B+1)} &= \rho \\
\frac{2N - N}{2(B+1)} &= \rho \\
\frac{N}{2(B+1)} &= \rho \\
\frac{N}{2\rho} &= B+1 \\
B &= \frac{N}{2\rho} - 1.
\end{aligned}$$

where

$$\rho = \begin{cases} \sum_{p=0}^{\frac{N-2}{2}} \frac{1}{2} - \frac{2}{S-2p+1}, & \text{when } N \text{ is even} \\ \sum_{p=0}^{\frac{N-3}{2}} \frac{1}{2} - \frac{2}{S-2p}, & \text{when } N \text{ is odd.} \end{cases}$$

By the same reasoning, the buyer will prefer a forward auction if and only if $D_N^{FA} > D_N^{RFA}$, i.e., if and only if

$$\begin{aligned}
\frac{N}{B(B+1)} &> \frac{1}{B} \left(\frac{N}{2(B+1)} + \rho \right), \\
\frac{N}{B+1} &> \frac{N}{2(B+1)} + \rho, \\
\frac{N}{B+1} - \frac{N}{2(B+1)} &> \rho, \\
\frac{2N - N}{2(B+1)} &> \rho \\
\frac{N}{2(B+1)} &> \rho.
\end{aligned}$$

Because we divide by ρ to simplify this expression with respect to B , the key now is the sign of ρ . If $\rho > 0$, we can continue as above and conclude that the buyer will prefer a forward auction if and only if $B < \frac{N}{2\rho} - 1$. However, if $\rho < 0$, the sign of the inequality will switch when we divide both sides by ρ , and we conclude that the buyer will prefer a forward auction if and only if $B > \frac{N}{2\rho} - 1$.

To determine the sign of ρ , we use the standard assumptions that $S \geq N + 1$ and $N \geq 2$ (so that competition exists in every auction). We begin with the case that N is even.

Case 1: $S \geq 5$. In this case,

$$\begin{aligned}\rho &= \sum_{p=0}^{\frac{N-2}{2}} \frac{1}{2} - \frac{2}{S-2p+1} \text{ when } N \text{ is even} \\ &= \left(\frac{1}{2} - \frac{2}{S-(N-2)+1}\right) + \left(\frac{1}{2} - \frac{2}{S-(N-4)+1}\right) + \sum_{p=0}^{\frac{N-6}{2}} \left(\frac{1}{2} - \frac{2}{S-2p+1}\right).\end{aligned}$$

The terms in the summation decrease as the index increases since $\frac{1}{2} - \frac{2}{S+1} > \frac{1}{2} - \frac{2}{S} > \frac{1}{2} - \frac{2}{S-2} > \dots > \frac{1}{2} - \frac{2}{S-N+7}$. So the lowest value in the summation will be $\frac{1}{2} - \frac{2}{S-N+7}$. In addition, the lowest value of S will yield the lowest value for each difference (i.e., $\frac{1}{2} - \frac{2}{S-p+1}$). Since $S \geq N + 1$,

$$\rho \geq \left(\frac{1}{2} - \frac{2}{4}\right) + \left(\frac{1}{2} - \frac{2}{6}\right) + \sum_{p=0}^{N-6} \left(\frac{1}{2} - \frac{2}{8}\right).$$

Each of the terms in the summation is positive (since $\frac{1}{2} > \frac{2}{8}$) and the two terms outside the summation are $\frac{1}{2} + \frac{1}{2} - \frac{2}{4} - \frac{2}{6} = \frac{1}{2} - \frac{1}{3} > 0$; therefore, $\rho > 0$ and the buyer will prefer a forward auction if and only if $B < \frac{N}{2\rho} - 1$.

We continue in this case and consider the sign of ρ when N is odd. When N is odd,

$$\begin{aligned}\rho &= \sum_{p=0}^{\frac{N-3}{2}} \frac{1}{2} - \frac{2}{S-2p} \\ &= \left(\frac{1}{2} - \frac{2}{S-(N-3)}\right) + \left(\frac{1}{2} - \frac{2}{S-(N-5)}\right) + \sum_{p=0}^{\frac{N-7}{2}} \left(\frac{1}{2} - \frac{2}{S-2p}\right).\end{aligned}$$

The terms in the summation decrease as the index increases since $\frac{1}{2} - \frac{2}{S+1} > \frac{1}{2} - \frac{2}{S} > \frac{1}{2} - \frac{2}{S-2} > \dots > \frac{1}{2} - \frac{2}{S-N+7}$. So the lowest value in the summation will be $\frac{1}{2} - \frac{2}{S-N+7}$. In addition, the lowest value of S will yield the lowest value for each difference (i.e., $\frac{1}{2} - \frac{2}{S-p+1}$). Since $S \geq N + 1$,

$$\rho \geq \left(\frac{1}{2} - \frac{2}{4}\right) + \left(\frac{1}{2} - \frac{2}{6}\right) + \sum_{p=0}^{N-6} \left(\frac{1}{2} - \frac{2}{8}\right).$$

Each of the terms in the summation is positive (since $\frac{1}{2} > \frac{2}{8}$) and the two terms outside the summation are $\frac{1}{2} + \frac{1}{2} - \frac{2}{4} - \frac{2}{6} = \frac{1}{2} - \frac{1}{3} > 0$; therefore, $\rho > 0$ and the buyer will prefer a forward auction if and only if $B < \frac{N}{2\rho} - 1$.

Case 2: $S = 4$ and $N = 2$. If $N = 2$ then $\rho = \frac{1}{2} - \frac{2}{5} > 0$. In this case the buyer will prefer a forward auction whenever $B < \frac{N}{2\rho} - 1$.

Case 3: $S = N + 1$ and $N \in \{2, 3\}$. This case has two subcases. If $S = 3$ and $N = 2$, then $\rho = \frac{1}{2} - \frac{2}{4} = 0$. If $S = 4$ and $N = 3$, then $\rho = \frac{1}{2} - \frac{2}{4} = 0$. Therefore, we cannot divide by ρ . We return to the initial inequality, which states that the buyer will prefer the forward auction if and only if

$$\begin{aligned}
D_N^{FA} &> D_N^{RFA} \\
\frac{N}{B(B+1)} &> \frac{1}{B} \left(\frac{N}{2(B+1)} + \rho \right), \\
\frac{N}{B+1} &> \frac{N}{2(B+1)} + \rho, \\
\frac{N}{B+1} - \frac{N}{2(B+1)} &> \rho, \\
\frac{2N - N}{2(B+1)} &> \rho \\
\frac{N}{2(B+1)} &> \rho
\end{aligned}$$

Since both B and N must be positive, this inequality must hold; therefore, the buyer will prefer a forward auction in this case for any value of B . ■

The expected supplier profit functions integrated with respect to the private valuations are as follows:

Reverse Auction

$$\begin{aligned}
P_N^{RA} &= \int_0^1 \left(\frac{(1 - v_N)^S}{S} + \frac{N - 1}{S(S - N + 1)} \right) dv_N \\
&= \frac{1}{S(S + 1)} + \frac{N - 1}{S(S - N + 1)}
\end{aligned}$$

Forward Auction

$$\begin{aligned}
P_N^{FA} &= \int_0^1 \left(\frac{\frac{B-1}{B+1} - v_N}{S} + \left(\frac{B-1}{B+1} - \frac{1}{2} \right) \alpha \right) dv_N \\
&= \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S} + \left(\frac{B-1}{B+1} - \frac{1}{2} \right) \alpha,
\end{aligned}$$

where

$$\alpha = \sum_{p=0}^{N-2} \left(\prod_{q=0}^p \left(1 - \frac{1}{S - q} \right) \right) \frac{1}{S - p - 1}.$$

Alternating Auction

$$\begin{aligned}
P_N^{RFA} &= \int_0^1 \left(\frac{(1-v_N)^S}{S} + \frac{1}{S(S+1)} + \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S-1} + \left(\frac{B-1}{B+1} - \frac{1}{2} \right) * \tau + \kappa \right) dv_N \\
&= \frac{1}{S(S+1)} + \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S-1} + \left(\frac{B-1}{B+1} - \frac{1}{2} \right) * \tau + \kappa,
\end{aligned}$$

where

$$\begin{aligned}
\tau &= \sum_{p=1}^{\frac{N-2}{2}} \left(\prod_{q=1}^p \left(1 - \frac{1}{S-2q+1} \right) \right) \frac{1}{S-2p-1} \quad \text{when } N \text{ is even} \\
&= \sum_{p=0}^{\frac{N-3}{2}} \left(\prod_{q=0}^p \left(1 - \frac{1}{S-2q} \right) \right) \frac{1}{S-2p-2} \quad \text{when } N \text{ is odd} \\
\kappa &= \sum_{p=1}^{\frac{N-2}{2}} \left(\prod_{q=1}^p \left(1 - \frac{1}{S-2q+1} \right) \right) \frac{1}{(S-2p)(S-2p+1)} \quad \text{when } N \text{ is even} \\
&= \sum_{p=0}^{\frac{N-3}{2}} \left(\prod_{q=0}^p \left(1 - \frac{1}{S-2q} \right) \right) \frac{1}{(S-2p)(S-2p-1)} \quad \text{when } N \text{ is odd.}
\end{aligned}$$

Supplier Preference

In this section, we compare the N -period expected profit to the supplier under each auction sequence.

Proposition 3.5.4(Supplier Reverse Auction vs. Forward Auction) A profit-maximizing supplier will be indifferent between a reverse auction and a forward auction when

$$B = \frac{1+\gamma}{1-\gamma},$$

where

$$\gamma = \frac{2SN}{(S+1)(S-N+1)(1+\alpha S)} + 1,$$

and where

$$\alpha = \sum_{p=0}^{N-2} \left(\prod_{q=0}^p \left(1 - \frac{1}{S-q} \right) \right) \frac{1}{S-p-1},$$

Under the standard assumptions that $S \geq N+1$ and $N \geq 2$, we have that the buyer will prefer the reverse auction to the forward auction if and only if $B > \frac{1+\gamma}{1-\gamma}$.

Proof of Proposition 3.5.4

Because the supplier looks to maximize expected profit, she will be indifferent whenever $P_N^{RA} = P_N^{FA}$. As shown above, this is equivalent to

$$\begin{aligned}
P_N^{RA} &= P_N^{FA} \\
\frac{1}{S(S+1)} + \frac{N-1}{S(S-N+1)} &= \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S} + (\frac{B-1}{B+1} - \frac{1}{2})\alpha \\
\frac{1}{S+1} + \frac{N-1}{S-N+1} &= \frac{B-1}{B+1} - \frac{1}{2} + (\frac{B-1}{B+1} - \frac{1}{2})\alpha S \\
\frac{1}{S+1} + \frac{N-1}{S-N+1} + \frac{1}{2} &= \frac{B-1}{B+1} + \frac{B-1}{B+1}\alpha S - \frac{1}{2}\alpha S \\
\frac{1}{S+1} + \frac{N-1}{S-N+1} + \frac{1}{2} + \frac{1}{2}\alpha S &= \frac{B-1}{B+1} + \frac{B-1}{B+1}\alpha S \\
\frac{1}{S+1} + \frac{N-1}{S-N+1} + \frac{1}{2} + \frac{1}{2}\alpha S &= \frac{B-1}{B+1}(1 + \alpha S) \\
\frac{1}{S+1} + \frac{N-1}{S-N+1} + \frac{1}{2} + \frac{1}{2}\alpha S &= \frac{B-1}{B+1}(1 + \alpha S) \\
\frac{(S-N+1) + (S+1)(N-1)}{(S+1)(S-N+1)} + \frac{1}{2} + \frac{1}{2}\alpha S &= \frac{B-1}{B+1}(1 + \alpha S) \\
\frac{S-N+1 + SN + N - S - 1}{(S+1)(S-N+1)} + \frac{1}{2} + \frac{1}{2}\alpha S &= \frac{B-1}{B+1}(1 + \alpha S) \\
\frac{SN}{(S+1)(S-N+1)} + \frac{1}{2} + \frac{1}{2}\alpha S &= \frac{B-1}{B+1}(1 + \alpha S).
\end{aligned}$$

α simplifies as follows:

$$\begin{aligned}
\alpha &= \sum_{p=0}^{N-2} \left(\prod_{q=0}^p \left(1 - \frac{1}{S-q} \right) \right) \frac{1}{S-p-1} \\
&= \sum_{p=0}^{N-2} \left(\frac{S-1}{S} \right) \frac{1}{S-1} + \left(\frac{S-1}{S} \right) \left(\frac{S-2}{S-1} \right) \frac{1}{S-2} + \left(\frac{S-1}{S} \right) \left(\frac{S-2}{S-1} \right) \left(\frac{S-3}{S-2} \right) \frac{1}{S-3} \\
&+ \dots + \left(\frac{S-1}{S} \right) \left(\frac{S-2}{S-1} \right) \dots \left(\frac{S-N+1}{S-N} \right) \frac{1}{S-N+1} \\
&= \sum_{p=0}^{N-2} \frac{1}{S} = \frac{N-1}{S}.
\end{aligned}$$

Therefore, using the standard assumptions that $S \geq N+1$ and $N \geq 2$, $\alpha > 0$ and we can divide both sides of the equality by $1 + \alpha S$ as follows:

$$\begin{aligned}
\frac{\frac{2SN}{(S+1)(S-N+1)} + 1 + \alpha S}{1 + \alpha S} &= \frac{B-1}{B+1} \\
\frac{2SN}{(S+1)(S-N+1)(1 + \alpha S)} + \frac{1 + \alpha S}{1 + \alpha S} &= \frac{B-1}{B+1} \\
\frac{2SN}{(S+1)(S-N+1)(1 + \alpha S)} + 1 &= \frac{B-1}{B+1}
\end{aligned}$$

Simplifying this equality with respect to B and making the substitution

$\gamma = \frac{2SN}{(S+1)(S-N+1)(1+\alpha S)} + 1$ gives

$$\begin{aligned}\gamma &= \frac{B-1}{B+1} \\ \gamma B + \gamma &= B-1 \\ \gamma + 1 &= B - \gamma B \\ \frac{1+\gamma}{1-\gamma} &= B.\end{aligned}$$

By the same reasoning, the supplier prefers the reverse auction to the forward auction when the expected profit of the reverse auction exceeds that of the forward auction. This is expressed as follows:

$$\begin{aligned}P_N^{RA} &> P_N^{FA} \\ \frac{1}{S(S+1)} + \frac{N-1}{S(S-N+1)} &> \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S} + (\frac{B-1}{B+1} - \frac{1}{2})\alpha \\ \frac{1}{S+1} + \frac{N-1}{S-N+1} &> \frac{B-1}{B+1} - \frac{1}{2} + (\frac{B-1}{B+1} - \frac{1}{2})\alpha S \\ \frac{1}{S+1} + \frac{N-1}{S-N+1} + \frac{1}{2} &> \frac{B-1}{B+1} + \frac{B-1}{B+1}\alpha S - \frac{1}{2}\alpha S \\ \frac{1}{S+1} + \frac{N-1}{S-N+1} + \frac{1}{2} + \frac{1}{2}\alpha S &> \frac{B-1}{B+1} + \frac{B-1}{B+1}\alpha S \\ \frac{1}{S+1} + \frac{N-1}{S-N+1} + \frac{1}{2} + \frac{1}{2}\alpha S &> \frac{B-1}{B+1}(1+\alpha S) \\ \frac{1}{S+1} + \frac{N-1}{S-N+1} + \frac{1}{2} + \frac{1}{2}\alpha S &> \frac{B-1}{B+1}(1+\alpha S) \\ \frac{(S-N+1) + (S+1)(N-1)}{(S+1)(S-N+1)} + \frac{1}{2} + \frac{1}{2}\alpha S &> \frac{B-1}{B+1}(1+\alpha S) \\ \frac{S-N+1 + SN + N - S - 1}{(S+1)(S-N+1)} + \frac{1}{2} + \frac{1}{2}\alpha S &> \frac{B-1}{B+1}(1+\alpha S) \\ \frac{SN}{(S+1)(S-N+1)} + \frac{1}{2} + \frac{1}{2}\alpha S &> \frac{B-1}{B+1}(1+\alpha S)\end{aligned}$$

As shown above, $\alpha = \frac{N-1}{S}$. Using the standard assumptions that $S \geq N+1$ and $N \geq 2$, $\alpha > 0$ and we can divide both sides of the inequality by $1 + \alpha S$ without switching the sign as follows:

$$\begin{aligned}\frac{\frac{2SN}{(S+1)(S-N+1)} + 1 + \alpha S}{1 + \alpha S} &> \frac{B-1}{B+1}, \\ \frac{2SN}{(S+1)(S-N+1)(1+\alpha S)} + \frac{1+\alpha S}{1+\alpha S} &> \frac{B-1}{B+1}, \\ \frac{2SN}{(S+1)(S-N+1)(1+\alpha S)} + 1 &> \frac{B-1}{B+1},\end{aligned}$$

Simplifying this inequality with respect to B and making the substitution

$\gamma = \frac{2SN}{(S+1)(S-N+1)(1+\alpha S)} + 1$ gives

$$\begin{aligned}\gamma &> \frac{B-1}{B+1}, \\ \gamma B + \gamma &> B-1, \\ \gamma + 1 &> B - \gamma B, \\ \frac{1+\gamma}{1-\gamma} &> B.\end{aligned}$$

Our standard assumptions that $S \geq N+1$ and $N \geq 2$ require that $\gamma > 1$, therefore when we divide by $1-\gamma$, the inequality switches. Therefore, we have that the supplier prefers the reverse auction to the forward auction when $B > \frac{1+\gamma}{1-\gamma}$. ■

Proposition 3.5.5 (Supplier Reverse Auction vs. Alternating Auction) A profit-maximizing supplier will be indifferent between a reverse auction and an alternating auction when

$$B = \frac{1+\varrho}{1-\varrho}$$

where

$$\varrho = \frac{\frac{N-1}{S(S-N+1)}(S-1) - \kappa(S-1) + \frac{1}{2} + \frac{1}{2}\tau(S-1)}{1 + \tau(S-1)}$$

and where

$$\tau = \begin{cases} \sum_{p=1}^{\frac{N-2}{2}} (\prod_{q=1}^p (1 - \frac{1}{S-2q+1})) \frac{1}{S-2p-1}, & \text{when } N \text{ is even} \\ \sum_{p=0}^{\frac{N-3}{2}} (\prod_{q=0}^p (1 - \frac{1}{S-2q})) \frac{1}{S-2p-2}, & \text{when } N \text{ is odd} \\ 0, & \text{when } N = 2. \end{cases}$$

and

$$\kappa = \begin{cases} \sum_{p=1}^{\frac{N-2}{2}} (\prod_{q=1}^p (1 - \frac{1}{S-2q+1})) \frac{1}{(S-2p)(S-2p+1)}, & \text{when } N \text{ is even} \\ \sum_{p=0}^{\frac{N-3}{2}} (\prod_{q=0}^p (1 - \frac{1}{S-2q})) \frac{1}{(S-2p)(S-2p-1)}, & \text{when } N \text{ is odd} \\ 0, & \text{when } N = 2. \end{cases}$$

Under the standard assumptions that $S \geq N+1$ and $N \geq 2$, the buyer will prefer the reverse auction to the alternating auction if and only if $B > \frac{1+\varrho}{1-\varrho}$.

Proof of Proposition 3.5.5

Because the supplier looks to maximize expected profit, she will be indifferent whenever

$P_N^{RA} = P_N^{RFA}$. As shown above, this is equivalent to

$$\begin{aligned}
P_N^{RA} &= P_N^{RFA}, \\
\frac{1}{S(S+1)} + \frac{N-1}{S(S-N+1)} &= \frac{1}{S(S+1)} + \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S-1} + (\frac{B-1}{B+1} - \frac{1}{2})\tau + \kappa, \\
\frac{N-1}{S(S-N+1)} &= \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S-1} + (\frac{B-1}{B+1} - \frac{1}{2})\tau + \kappa, \\
\frac{N-1}{S(S-N+1)} - \kappa &= \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S-1} + (\frac{B-1}{B+1} - \frac{1}{2})\tau, \\
\frac{(N-1)(S-1)}{S(S-N+1)} - \kappa(S-1) &= \frac{B-1}{B+1} - \frac{1}{2} + (\frac{B-1}{B+1} - \frac{1}{2})\tau(S-1), \\
\frac{(N-1)(S-1)}{S(S-N+1)} - \kappa(S-1) + \frac{1}{2} &= \frac{B-1}{B+1} + (\frac{B-1}{B+1} - \frac{1}{2})\tau(S-1), \\
\frac{(N-1)(S-1)}{S(S-N+1)} - \kappa(S-1) + \frac{1}{2} &= \frac{B-1}{B+1} + \frac{B-1}{B+1}\tau(S-1) - \frac{1}{2}\tau(S-1), \\
\frac{(N-1)(S-1)}{S(S-N+1)} - \kappa(S-1) + \frac{1}{2} + \frac{1}{2}\tau(S-1) &= \frac{B-1}{B+1}(1 + \tau(S-1)), \\
\frac{2(N-1)(S-1)}{S(S-N+1)} - 2\kappa(S-1) + 1 + \tau(S-1) &= \frac{B-1}{B+1}(1 + \tau(S-1)).
\end{aligned}$$

Because $\tau \geq 0$ for every S and N using the standard assumptions that $S \geq N+1$ and $N \geq 2$, we are able to divide both sides of the equality by $1 + \tau(S-1)$ to obtain:

$$\frac{B-1}{B+1} = \frac{\frac{2(N-1)(S-1)}{S(S-N+1)} - 2\kappa(S-1)}{1 + \tau(S-1)} + 1.$$

Simplifying this equality with respect to B and making the substitution

$$\varrho = \frac{\frac{2(N-1)(S-1)}{S(S-N+1)} - 2\kappa(S-1)}{1 + \tau(S-1)} + 1 \text{ gives}$$

$$\begin{aligned}
\frac{B-1}{B+1} &= \varrho \\
B-1 &= \varrho B + \varrho \\
B - \varrho B &= \varrho + 1 \\
B &= \frac{1 + \varrho}{1 - \varrho},
\end{aligned}$$

By the same reasoning, the supplier prefers the reverse auction to the alternating auction when the expected profit of the reverse auction exceeds that of the alternating auction. This

is expressed as follows:

$$\begin{aligned}
P_N^{RA} &> P_N^{RFA} \\
\frac{1}{S(S+1)} + \frac{N-1}{S(S-N+1)} &> \frac{1}{S(S+1)} + \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S-1} + (\frac{B-1}{B+1} - \frac{1}{2})\tau + \kappa, \\
\frac{N-1}{S(S-N+1)} &> \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S-1} + (\frac{B-1}{B+1} - \frac{1}{2})\tau + \kappa, \\
\frac{N-1}{S(S-N+1)} - \kappa &> \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S-1} + (\frac{B-1}{B+1} - \frac{1}{2})\tau, \\
\frac{(N-1)(S-1)}{S(S-N+1)} - \kappa(S-1) &> \frac{B-1}{B+1} - \frac{1}{2} + (\frac{B-1}{B+1} - \frac{1}{2})\tau(S-1), \\
\frac{(N-1)(S-1)}{S(S-N+1)} - \kappa(S-1) + \frac{1}{2} &> \frac{B-1}{B+1} + (\frac{B-1}{B+1} - \frac{1}{2})\tau(S-1), \\
\frac{(N-1)(S-1)}{S(S-N+1)} - \kappa(S-1) + \frac{1}{2} &> \frac{B-1}{B+1} + \frac{B-1}{B+1}\tau(S-1) - \frac{1}{2}\tau(S-1), \\
\frac{(N-1)(S-1)}{S(S-N+1)} - \kappa(S-1) + \frac{1}{2} + \frac{1}{2}\tau(S-1) &> \frac{B-1}{B+1}(1 + \tau(S-1)), \\
\frac{2(N-1)(S-1)}{S(S-N+1)} - 2\kappa(S-1) + 1 + \tau(S-1) &> \frac{B-1}{B+1}(1 + \tau(S-1)).
\end{aligned}$$

Because $\tau \geq 0$ for every S and N using the standard assumptions that $S \geq N+1$ and $N \geq 2$, we are able to divide both sides of the inequality by $1 + \tau(S-1)$ to obtain

$$\frac{B-1}{B+1} < \frac{\frac{2(N-1)(S-1)}{S(S-N+1)} - 2\kappa(S-1)}{1 + \tau(S-1)} + 1.$$

Simplifying this inequality with respect to B and making the substitution

$$\varrho = \frac{\frac{2(N-1)(S-1)}{S(S-N+1)} - 2\kappa(S-1)}{1 + \tau(S-1)} + 1 \text{ gives}$$

$$\begin{aligned}
\frac{B-1}{B+1} &< \varrho \\
B-1 &< \varrho B + \varrho \\
B - \varrho B &< \varrho + 1 \\
B(1 - \varrho) &< \varrho + 1
\end{aligned}$$

To finish simplifying this inequality with respect to B we divide by $1 - \varrho$ so we must determine the sign of $1 - \varrho$. If $1 - \varrho > 0$, we can continue as above and conclude that the buyer will prefer a reverse auction if and only if $B < \frac{1+\varrho}{1-\varrho}$. However, if $1 - \varrho < 0$, the sign of the inequality will switch when we divide both sides by $1 - \varrho$, and we conclude that the buyer will prefer a reverse auction if and only if $B > \frac{1+\varrho}{1-\varrho}$.

To determine the sign of $1 - \varrho$, we determine whether $\varrho > 1$ as follows:

$$\begin{aligned}
\varrho &= \frac{\frac{2(N-1)(S-1)}{S(S-N+1)} - 2\kappa(S-1)}{1 + \tau(S-1)} + 1 \\
&= \frac{2(N-1)(S-1)}{S(S-N+1)(1 + \tau(S-1))} - \frac{2\kappa(S-1)}{1 + \tau(S-1)} + 1.
\end{aligned}$$

So $\varrho > 1$ when

$$\begin{aligned} \frac{2(N-1)(S-1)}{S(S-N+1)(1+\tau(S-1))} - \frac{2\kappa(S-1)}{1+\tau(S-1)} &> 0 \\ \frac{N-1}{S(S-N+1)} - \kappa &> 0. \end{aligned}$$

Because κ depends on whether N is even or odd, we consider this inequality for both cases of N .

Case 1: N is even. When N is even, $\varrho > 1$ when

$$\frac{N-1}{S(S-N+1)} - \sum_{p=1}^{\frac{N-2}{2}} \left(\prod_{q=1}^p \left(1 - \frac{1}{S-2q+1} \right) \right) \frac{1}{(S-2p)(S-2p+1)} > 0,$$

which is true when

$$\frac{N-1}{S(S-N+1)} > \sum_{p=1}^{\frac{N-2}{2}} \left(\prod_{q=1}^p \left(1 - \frac{1}{S-2q+1} \right) \right) \frac{1}{(S-2p)(S-2p+1)}. \quad (27)$$

The right side of the inequality can be simplified to

$$\sum_{p=1}^{\frac{N-2}{2}} \left(\prod_{q=1}^p \left(\frac{S-2q}{S-2q+1} \right) \right) \frac{1}{(S-2p)(S-2p+1)} < \sum_{p=1}^{\frac{N-2}{2}} \left(\prod_{q=1}^p (1) \right) \frac{1}{(S-2p)(S-2p+1)},$$

because each term in the product is less than one. We further simplify the summation by removing the multiple product terms of one to obtain

$$\sum_{p=1}^{\frac{N-2}{2}} \left(\prod_{q=1}^p (1) \right) \frac{1}{(S-2p)(S-2p+1)} = \sum_{p=1}^{\frac{N-2}{2}} \frac{1}{(S-2p)(S-2p+1)}.$$

Therefore,

$$\sum_{p=1}^{\frac{N-2}{2}} \left(\prod_{q=1}^p \left(\frac{S-2q}{S-2q+1} \right) \right) \frac{1}{(S-2p)(S-2p+1)} < \sum_{p=1}^{\frac{N-2}{2}} \frac{1}{(S-2p)(S-2p+1)}.$$

The right side of the inequality can be expressed as

$$\sum_{p=1}^{\frac{N-2}{2}} \frac{1}{(S-2p)(S-2p+1)} = \sum_{p=1}^{\frac{N-2}{2}} \left(\frac{1}{S-2p} + \frac{-1}{S-2p+1} \right).$$

When we reorder the terms and expand the sum we obtain

$$\begin{aligned} \sum_{p=1}^{\frac{N-2}{2}} \left(\frac{-1}{S-2p+1} + \frac{1}{S-2p} \right) &= \left(\frac{-1}{S-1} + \frac{1}{S-2} \right) + \left(\frac{-1}{S-3} + \frac{1}{S-4} \right) + \left(\frac{-1}{S-5} + \frac{1}{S-6} \right) \\ &\quad + \dots + \left(\frac{-1}{S-N+5} + \frac{1}{S-N+4} \right) + \left(\frac{-1}{S-N+3} + \frac{1}{S-N+2} \right). \end{aligned}$$

We regroup the terms to obtain

$$\begin{aligned} \sum_{p=1}^{\frac{N-2}{2}} \left(\frac{-1}{S-2p+1} + \frac{1}{S-2p} \right) &= \frac{-1}{S-1} + \left(\frac{1}{S-2} - \frac{1}{S-3} \right) + \left(\frac{1}{S-4} - \frac{1}{S-5} \right) + \left(\frac{1}{S-6} \right. \\ &\quad \left. - \dots - \frac{1}{S-N+5} \right) + \left(\frac{1}{S-N+4} - \frac{1}{S-N+3} \right) + \frac{1}{S-N+2}. \end{aligned}$$

Because all of the terms grouped in a set of parenthesis are of the form $\frac{1}{x} - \frac{1}{x-1} = \frac{(x-1)-x}{x(x-1)} = \frac{-1}{x(x-1)} < 0$, they are less than zero. Therefore,

$$\sum_{p=1}^{\frac{N-2}{2}} \left(\frac{-1}{S-2p+1} + \frac{1}{S-2p} \right) < \frac{-1}{S-1} + \frac{1}{S-N+2}.$$

The right side of this inequality simplifies as follows:

$$\begin{aligned} \frac{-1}{S-1} + \frac{1}{S-N+2} &= \frac{-(S-N+2) + (S-1)}{(S-1)(S-N+2)} \\ &= \frac{N-3}{(S-1)(S-N+2)}. \end{aligned}$$

It is true that

$$\frac{N-3}{(S-1)(S-N+2)} \leq \frac{N-3}{S(S-N+1)} \quad (28)$$

because $(S-1)(S-N+2) \geq S(S-N+1)$ for all $N \geq 2$, as follows:

$$\begin{aligned} (S-1)(S-N+2) - S(S-N+1) &= (S^2 - S - NS + N + 2S - 2) - (S^2 - NS + S) \\ &= N - 2. \end{aligned}$$

Since the sequence must include at least two auctions ($N \geq 2$), the difference is always nonnegative and the inequality holds. We compare the right side of the inequality on line (28) to the left side of the inequality on line (27) to obtain

$$\frac{N-1}{S(S-N+1)} > \frac{N-3}{S(S-N+1)},$$

because $N-1 > N-3$. By putting all of these inequalities and equations together, we conclude that the inequality on line (27) holds. Specifically,

$$\frac{N-1}{S(S-N+1)} - \sum_{p=1}^{\frac{N-2}{2}} \left(\prod_{q=1}^p \left(1 - \frac{1}{S-2q+1} \right) \right) \frac{1}{(S-2p)(S-2p+1)} > 0.$$

Therefore, when N is even, $\varrho > 1$ and the supplier prefers the reverse auction to the alternating auction if and only if $B > \frac{1+\varrho}{1-\varrho}$.

We now consider the case when N is odd.

Case 2: N is odd. In this case, using κ when N is odd, $\varrho > 1$ when

$$\frac{N-1}{S(S-N+1)} - \sum_{p=0}^{\frac{N-3}{2}} \left(\prod_{q=0}^p \left(1 - \frac{1}{S-2q} \right) \right) \frac{1}{(S-2p)(S-2p-1)} > 0,$$

which occurs when

$$\frac{N-1}{S(S-N+1)} > \sum_{p=0}^{\frac{N-3}{2}} \left(\prod_{q=0}^p \left(1 - \frac{1}{S-2q} \right) \right) \frac{1}{(S-2p)(S-2p-1)}. \quad (29)$$

The right side of the inequality can be simplified to

$$\sum_{p=0}^{\frac{N-3}{2}} \left(\prod_{q=0}^p \left(\frac{S-2q-1}{S-2q} \right) \right) \frac{1}{(S-2p)(S-2p-1)} < \sum_{p=0}^{\frac{N-3}{2}} \left(\prod_{q=0}^p (1) \right) \frac{1}{(S-2p)(S-2p-1)},$$

because each term in the product is less than one. We further simplify the summation by removing the multiple product terms of one to obtain

$$\sum_{p=0}^{\frac{N-3}{2}} \left(\prod_{q=0}^p (1) \right) \frac{1}{(S-2p)(S-2p-1)} = \sum_{p=0}^{\frac{N-3}{2}} \frac{1}{(S-2p)(S-2p-1)}.$$

Therefore,

$$\sum_{p=0}^{\frac{N-3}{2}} \left(\prod_{q=0}^p \left(\frac{S-2q-1}{S-2q} \right) \right) \frac{1}{(S-2p)(S-2p-1)} < \sum_{p=0}^{\frac{N-3}{2}} \frac{1}{(S-2p)(S-2p-1)}.$$

The right side of the inequality can be expressed as

$$\sum_{p=0}^{\frac{N-3}{2}} \frac{1}{(S-2p)(S-2p-1)} = \sum_{p=0}^{\frac{N-3}{2}} \left(\frac{-1}{S-2p} + \frac{1}{S-2p-1} \right).$$

When we expand the sum we obtain

$$\begin{aligned} \sum_{p=0}^{\frac{N-3}{2}} \left(\frac{-1}{S-2p} + \frac{1}{S-2p-1} \right) &= \left(\frac{-1}{S} + \frac{1}{S-1} \right) + \left(\frac{-1}{S-2} + \frac{1}{S-3} \right) + \left(\frac{-1}{S-4} + \frac{1}{S-5} \right) \\ &\quad + \dots + \left(\frac{-1}{S-N+5} + \frac{1}{S-N+4} \right) + \left(\frac{-1}{S-N+3} + \frac{1}{S-N+2} \right). \end{aligned}$$

When we regroup the terms we obtain

$$\begin{aligned} \sum_{p=0}^{\frac{N-3}{2}} \left(\frac{-1}{S-2p} + \frac{1}{S-2p-1} \right) &= \frac{-1}{S} + \left(\frac{1}{S-1} + \frac{-1}{S-2} \right) + \left(\frac{1}{S-3} + \frac{-1}{S-4} \right) + \left(\frac{1}{S-5} \right. \\ &\quad \left. + \dots + \frac{-1}{S-N+5} \right) + \left(\frac{1}{S-N+4} + \frac{-1}{S-N+3} \right) + \frac{1}{S-N+2}. \end{aligned}$$

Because all of the terms grouped in a set of parenthesis are of the form $\frac{1}{x} - \frac{1}{x-1} = \frac{(x-1)-x}{x(x-1)} = \frac{-1}{x(x-1)} < 0$, they are less than zero. Therefore,

$$\sum_{p=0}^{\frac{N-3}{2}} \left(\frac{-1}{S-2p} + \frac{1}{S-2p-1} \right) < \frac{-1}{S} + \frac{1}{S-N+2}.$$

The right side of this inequality simplifies as follows:

$$\begin{aligned} \frac{-1}{S} + \frac{1}{S-N+2} &= \frac{-(S-N+2)+S}{S(S-N+2)} \\ &= \frac{N-2}{S(S-N+2)}. \end{aligned}$$

It is true that

$$\frac{N-2}{S(S-N+2)} \leq \frac{N-2}{S(S-N+1)} \quad (30)$$

because $(S - N + 2) > (S - N + 1)$. We compare the right side of the inequality on line (30) to the left side of the inequality on line (29) to obtain

$$\frac{N-1}{S(S-N+1)} > \frac{N-2}{S(S-N+1)},$$

because $N-1 > N-2$. By putting all of these inequalities and equations together, we conclude that the inequality on line (29) holds. Specifically,

$$\frac{N-1}{S(S-N+1)} - \sum_{p=0}^{\frac{N-3}{2}} \left(\prod_{q=0}^p \left(1 - \frac{1}{S-2q} \right) \right) \frac{1}{(S-2p)(S-2p-1)}.$$

Therefore, when N is odd, $\varrho > 1$ and the supplier prefers the reverse auction to the alternating auction if and only if $B > \frac{1+\varrho}{1-\varrho}$. ■

Proposition 3.5.6 (Supplier Forward Auction vs. Alternating Auction) A profit-maximizing supplier will be indifferent between a forward auction and an alternating auction when

$$B = \frac{1+\eta}{1-\eta},$$

where

$$\eta = \frac{\frac{2(S-1)}{S+1} + 2\kappa S(S-1)}{(\alpha - \tau)S(S-1) - 1} + 1,$$

where

$$\alpha = \sum_{p=0}^{N-2} \left(\prod_{q=0}^p \left(1 - \frac{1}{S-q} \right) \right) \frac{1}{S-p-1},$$

and where

$$\tau = \begin{cases} \sum_{p=1}^{\frac{N-2}{2}} \left(\prod_{q=1}^p \left(1 - \frac{1}{S-2q+1} \right) \right) \frac{1}{S-2p-1}, & \text{when } N \text{ is even} \\ \sum_{p=0}^{\frac{N-3}{2}} \left(\prod_{q=0}^p \left(1 - \frac{1}{S-2q} \right) \right) \frac{1}{S-2p-2}, & \text{when } N \text{ is odd} \\ 0, & \text{when } N = 2 \end{cases}$$

and

$$\kappa = \begin{cases} \sum_{p=1}^{\frac{N-2}{2}} \left(\prod_{q=1}^p \left(1 - \frac{1}{S-2q+1} \right) \right) \frac{1}{(S-2p)(S-2p+1)}, & \text{when } N \text{ is even} \\ \sum_{p=0}^{\frac{N-3}{2}} \left(\prod_{q=0}^p \left(1 - \frac{1}{S-2q} \right) \right) \frac{1}{(S-2p)(S-2p-1)}, & \text{when } N \text{ is odd.} \\ 0, & \text{when } N = 2. \end{cases}$$

Under the standard assumptions that $S \geq N+1$ and $N \geq 2$, the supplier will prefer the forward auction to the alternating auction if and only if $B > \frac{1+\eta}{1-\eta}$.

Proof of Proposition 3.5.6

A profit-maximizing supplier will be indifferent between a forward auction and an alternating auction when

$$B = \frac{1 + \eta}{1 - \eta},$$

where

$$\eta = \frac{\frac{2(S-1)}{S+1} + 2\kappa S(S-1)}{(\alpha - \tau)S(S-1) - 1} + 1,$$

where

$$\alpha = \sum_{p=0}^{N-2} \left(\prod_{q=0}^p \left(1 - \frac{1}{S-q} \right) \right) \frac{1}{S-p-1},$$

and where

$$\tau = \begin{cases} \sum_{p=1}^{\frac{N-2}{2}} \left(\prod_{q=1}^p \left(1 - \frac{1}{S-2q+1} \right) \right) \frac{1}{S-2p-1}, & \text{when } N \text{ is even} \\ \sum_{p=0}^{\frac{N-3}{2}} \left(\prod_{q=0}^p \left(1 - \frac{1}{S-2q} \right) \right) \frac{1}{S-2p-2}, & \text{when } N \text{ is odd} \\ 0, & \text{when } N = 2 \end{cases}$$

and

$$\kappa = \begin{cases} \sum_{p=1}^{\frac{N-2}{2}} \left(\prod_{q=1}^p \left(1 - \frac{1}{S-2q+1} \right) \right) \frac{1}{(S-2p)(S-2p+1)}, & \text{when } N \text{ is even} \\ \sum_{p=0}^{\frac{N-3}{2}} \left(\prod_{q=0}^p \left(1 - \frac{1}{S-2q} \right) \right) \frac{1}{(S-2p)(S-2p-1)}, & \text{when } N \text{ is odd.} \\ 0, & \text{when } N = 2. \end{cases}$$

Under the standard assumptions that $S \geq N + 1$ and $N \geq 2$, the supplier will prefer the forward auction to the alternating auction if and only if $B > \frac{1+\eta}{1-\eta}$.

Proof of Proposition 3.5.6

Because the supplier looks to maximize expected profit, she will be indifferent whenever

$P_N^{FA} = P_N^{RFA}$. This is equivalent to

$$\begin{aligned}
P_N^{FA} &= P_N^{RFA}, \\
\frac{\frac{B-1}{B+1} - \frac{1}{2}}{S} + (\frac{B-1}{B+1} - \frac{1}{2})\alpha &= \frac{1}{S(S+1)} + \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S-1} + (\frac{B-1}{B+1} - \frac{1}{2})\tau + \kappa, \\
\frac{\frac{B-1}{B+1} - \frac{1}{2}}{S} + (\frac{B-1}{B+1} - \frac{1}{2})\alpha - \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S-1} - (\frac{B-1}{B+1} - \frac{1}{2})\tau &= \frac{1}{S(S+1)} + \kappa, \\
\frac{\frac{B-1}{B+1} - \frac{1}{2}}{S} - \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S-1} + (\frac{B-1}{B+1} - \frac{1}{2})(\alpha - \tau) &= \frac{1}{S(S+1)} + \kappa, \\
(\frac{B-1}{B+1} - \frac{1}{2})(S-1) + (\frac{B-1}{B+1} - \frac{1}{2})(\alpha - \tau)S(S-1) & \\
- (\frac{B-1}{B+1} - \frac{1}{2})S &= \frac{S-1}{S+1} + \kappa S(S-1), \\
(\frac{B-1}{B+1} - \frac{1}{2})(\alpha - \tau)S(S-1) - (\frac{B-1}{B+1} - \frac{1}{2}) &= \frac{S-1}{S+1} + \kappa S(S-1), \\
(\frac{B-1}{B+1} - \frac{1}{2})\left((\alpha - \tau)S(S-1) - 1\right) &= \frac{S-1}{S+1} + \kappa S(S-1),
\end{aligned}$$

We next divide both sides of the equality by $(\alpha - \tau)S(S-1) - 1$ and later prove that it will never equal zero.

$$\begin{aligned}
\frac{B-1}{B+1} - \frac{1}{2} &= \frac{\frac{S-1}{S+1} + \kappa S(S-1)}{(\alpha - \tau)S(S-1) - 1}, \\
\frac{B-1}{B+1} &= \frac{\frac{S-1}{S+1} + \kappa S(S-1)}{(\alpha - \tau)S(S-1) - 1} + \frac{1}{2}, \\
\frac{B-1}{B+1} &= \frac{\frac{2(S-1)}{S+1} + 2\kappa S(S-1)}{(\alpha - \tau)S(S-1) - 1} + 1.
\end{aligned}$$

Simplifying this equality with respect to B and making the substitution

$$\eta = \frac{\frac{2(S-1)}{S+1} + 2\kappa S(S-1)}{(\alpha - \tau)S(S-1) - 1} + 1 \text{ gives}$$

$$\begin{aligned}
\eta &= \frac{B-1}{B+1} \\
\eta B + \eta &= B-1 \\
\eta + 1 &= B - \eta B \\
B &= \frac{1+\eta}{1-\eta},
\end{aligned}$$

where

$$\alpha = \sum_{p=0}^{N-2} \left(\prod_{q=0}^p \left(1 - \frac{1}{S-q} \right) \right) \frac{1}{S-p-1},$$

and where

$$\tau = \begin{cases} \sum_{p=1}^{\frac{N-2}{2}} \left(\prod_{q=1}^p \left(1 - \frac{1}{S-2q+1} \right) \right) \frac{1}{S-2p-1}, & \text{when } N \text{ is even} \\ \sum_{p=0}^{\frac{N-3}{2}} \left(\prod_{q=0}^p \left(1 - \frac{1}{S-2q} \right) \right) \frac{1}{S-2p-2}, & \text{when } N \text{ is odd} \\ 0, & \text{when } N = 2 \end{cases}$$

and

$$\kappa = \begin{cases} \sum_{p=1}^{\frac{N-2}{2}} (\prod_{q=1}^p (1 - \frac{1}{S-2q+1})) \frac{1}{(S-2p)(S-2p+1)}, & \text{when } N \text{ is even} \\ \sum_{p=0}^{\frac{N-3}{2}} (\prod_{q=0}^p (1 - \frac{1}{S-2q})) \frac{1}{(S-2p)(S-2p-1)}, & \text{when } N \text{ is odd.} \\ 0, & \text{when } N = 2. \end{cases}$$

α simplifies as follows:

$$\begin{aligned} \alpha &= \sum_{p=0}^{N-2} (\prod_{q=0}^p (1 - \frac{1}{S-q})) \frac{1}{S-p-1} \\ &= \sum_{p=0}^{N-2} (\frac{S-1}{S}) \frac{1}{S-1} + (\frac{S-1}{S}) (\frac{S-2}{S-1}) \frac{1}{S-2} + (\frac{S-1}{S}) (\frac{S-2}{S-1}) (\frac{S-3}{S-2}) \frac{1}{S-3} \\ &\quad + \dots + (\frac{S-1}{S}) (\frac{S-2}{S-1}) \dots (\frac{S-N+1}{S-N}) \frac{1}{S-N+1} \\ &= \sum_{p=0}^{N-2} \frac{1}{S} = \frac{N-1}{S}. \end{aligned} \tag{31}$$

By the same reasoning, the supplier prefers the forward auction to the alternating auction when the expected profit of the forward auction exceeds that of the alternating auction. This is expressed as follows:

$$\begin{aligned} P_N^{FA} &> P_N^{RFA}, \\ \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S} + (\frac{B-1}{B+1} - \frac{1}{2})\alpha &> \frac{1}{S(S+1)} + \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S-1} + (\frac{B-1}{B+1} - \frac{1}{2})\tau + \kappa, \\ \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S} + (\frac{B-1}{B+1} - \frac{1}{2})\alpha - \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S-1} - (\frac{B-1}{B+1} - \frac{1}{2})\tau &> \frac{1}{S(S+1)} + \kappa, \\ \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S} - \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S-1} + (\frac{B-1}{B+1} - \frac{1}{2})(\alpha - \tau) &> \frac{1}{S(S+1)} + \kappa, \\ (\frac{B-1}{B+1} - \frac{1}{2})(S-1) + (\frac{B-1}{B+1} - \frac{1}{2})(\alpha - \tau)S(S-1) &> \frac{S-1}{S+1} + \kappa S(S-1), \\ -(\frac{B-1}{B+1} - \frac{1}{2})S &> \frac{S-1}{S+1} + \kappa S(S-1), \\ (\frac{B-1}{B+1} - \frac{1}{2})(\alpha - \tau)S(S-1) - (\frac{B-1}{B+1} - \frac{1}{2}) &> \frac{S-1}{S+1} + \kappa S(S-1), \\ (\frac{B-1}{B+1} - \frac{1}{2})((\alpha - \tau)S(S-1) - 1) &> \frac{S-1}{S+1} + \kappa S(S-1). \end{aligned}$$

We next divide both sides of the inequality by $(\alpha - \tau)S(S-1) - 1$ and later prove that it is never equal to zero. If $(\alpha - \tau)S(S-1) - 1 > 0$, the inequality sign is unchanged and

$$\begin{aligned} \frac{B-1}{B+1} - \frac{1}{2} &> \frac{\frac{S-1}{S+1} + \kappa S(S-1)}{(\alpha - \tau)S(S-1) - 1}, \\ \frac{B-1}{B+1} &> \frac{\frac{S-1}{S+1} + \kappa S(S-1)}{(\alpha - \tau)S(S-1) - 1} + \frac{1}{2}, \\ \frac{B-1}{B+1} &> \frac{\frac{2(S-1)}{S+1} + 2\kappa S(S-1)}{(\alpha - \tau)S(S-1) - 1} + 1. \end{aligned}$$

If $(\alpha - \tau)S(S-1) - 1$ is negative, then the sign of the inequality switches. We later prove whether $(\alpha - \tau)S(S-1) - 1$ is positive or negative.

Simplifying this inequality with respect to B and making the substitution

$$\eta = \frac{\frac{2(S-1)}{S+1} + 2\kappa S(S-1)}{(\alpha-\tau)S(S-1)-1} + 1 \text{ gives}$$

$$\begin{aligned} \frac{B-1}{B+1} &> \eta, \\ B-1 &> \eta B + \eta, \\ B - \eta B &> \eta + 1 \\ B &> \frac{1+\eta}{1-\eta}, \end{aligned}$$

where α , κ , and τ are as defined above.

Because we divide by $1 - \eta$ to simplify this expression with respect to B , the key now is determining the sign of $1 - \eta$. If $1 - \eta > 0$, we can continue as above and conclude that the supplier will prefer a forward auction if and only if $B > \frac{1+\eta}{1-\eta}$. However, if $1 - \eta < 0$, the sign of the inequality will switch when we divide both sides by $1 - \eta$, and we conclude that the supplier will prefer a forward auction if and only if $B < \frac{1+\eta}{1-\eta}$.

To determine the sign of $1 - \eta$, since the numerator of $1 - \eta$ is positive, we determine whether the denominator is also positive. Specifically, we will check the sign of $(\alpha - \tau)S(S - 1) - 1$.

Using the simplification of α as shown in (31), the left side of this inequality can be expressed as

$$\begin{aligned} (\alpha - \tau)S(S - 1) - 1 &= \left(\frac{N-1}{S} - \tau\right)S(S - 1) - 1 \\ &= \frac{(N-1)S(S-1)}{S} - (\tau)S(S-1) - 1 \\ &= (N-1)(S-1) - (\tau)S(S-1) - 1 \\ &= NS - S - N + 1 - (\tau)S(S-1) - 1 \\ &= NS - S - N - (\tau)S(S-1). \end{aligned}$$

Therefore, to determine if the denominator of $1 - \eta$ is positive, we need to check if $NS - S - N > (\tau)S(S - 1)$, which can also be stated as $\frac{NS-S-N}{S(S-1)} > \tau$.

Because τ depends on whether N is even or odd, we consider this inequality for both cases of N . We will use the standard assumptions that $S \geq N + 1$ and $N \geq 2$.

Case 1: $N = 2$. When $N = 2$, $\tau = 0$ and $\frac{NS-S-N}{S(S-1)} = \frac{S-2}{S(S-1)} > 0$ since $S \geq N + 1 = 3$. In this case $\frac{NS-S-N}{S(S-1)} > \tau$ and $1 - \eta > 0$. Therefore, we can conclude that the supplier will prefer a forward auction if and only if $B > \frac{1+\eta}{1-\eta}$.

Case 2: N is even and $N \geq 4$. When N is even and $N \geq 4$, we determine if the inequality $\frac{NS-S-N}{S(S-1)} > \tau$ is valid for any S when $N \geq 4$. In this case, $\frac{NS-S-N}{S(S-1)} > \tau$ becomes

$$\frac{NS-S-N}{S(S-1)} > \sum_{p=1}^{\frac{N-2}{2}} \left(\prod_{q=1}^p \left(1 - \frac{1}{S-2q+1} \right) \right) \frac{1}{S-2p-1}. \quad (32)$$

The left side of the inequality in (32) can be expressed as

$$\begin{aligned} \frac{NS-S-N}{S(S-1)} &= \frac{N-1}{S-1} - \frac{N}{S(S-1)} \\ &= \frac{1}{S-1} \left(N-1 - \frac{N}{S} \right) > \frac{1}{S-1} (N-2) \end{aligned}$$

because $S > N$. Therefore,

$$\frac{NS-S-N}{S(S-1)} > \frac{1}{S-1} (N-2). \quad (33)$$

The right side of the inequality in (32) can be expanded as follows:

$$\begin{aligned} \sum_{p=1}^{\frac{N-2}{2}} \left(\prod_{q=1}^p \left(1 - \frac{1}{S-2q+1} \right) \right) \frac{1}{S-2p-1} &= \left(\frac{S-2}{S-1} \right) \left(\frac{1}{S-3} \right) + \left(\frac{S-2}{S-1} \right) \left(\frac{S-4}{S-3} \right) \left(\frac{1}{S-5} \right) \\ &+ \dots + \left(\frac{S-2}{S-1} \right) \left(\frac{S-4}{S-3} \right) \dots \left(\frac{S-N+2}{S-N+3} \right) \left(\frac{1}{S-N+1} \right). \end{aligned}$$

We regroup the terms to obtain

$$\begin{aligned} \sum_{p=1}^{\frac{N-2}{2}} \left(\prod_{q=1}^p \left(1 - \frac{1}{S-2q+1} \right) \right) \frac{1}{S-2p-1} &= \frac{1}{S-1} \left[\left(\frac{S-2}{S-3} \right) + \left(\frac{S-2}{S-3} \right) \left(\frac{S-4}{S-5} \right) \right. \\ &\left. + \dots + \left(\frac{S-2}{S-3} \right) \left(\frac{S-4}{S-5} \right) \dots \left(\frac{S-N+2}{S-N+1} \right) \right]. \end{aligned}$$

There are $\frac{N-2}{2}$ product terms within the brackets. We can express this as

$$\begin{aligned} &\frac{1}{S-1} \left[\left(\frac{S-2}{S-3} \right) + \left(\frac{S-2}{S-3} \right) \left(\frac{S-4}{S-5} \right) + \dots + \left(\frac{S-2}{S-3} \right) \left(\frac{S-4}{S-5} \right) \dots \left(\frac{S-N+2}{S-N+1} \right) \right] \\ &= \frac{1}{S-1} \sum_{p=1}^{\frac{N-2}{2}} \left(\prod_{q=1}^p \left(\frac{S-2q}{S-2q-1} \right) \right). \end{aligned}$$

Therefore,

$$\tau = \frac{1}{S-1} \sum_{p=1}^{\frac{N-2}{2}} \left(\prod_{q=1}^p \left(\frac{S-2q}{S-2q-1} \right) \right). \quad (34)$$

We combine (33) and (34), to show that the inequality in (32) holds if

$$\frac{NS-S-N}{S(S-1)} > \frac{1}{S-1} (N-2) > \frac{1}{S-1} \sum_{p=1}^{\frac{N-2}{2}} \left(\prod_{q=1}^p \left(\frac{S-2q}{S-2q-1} \right) \right) = \tau.$$

This is also expressed as

$$N - 2 > \sum_{p=1}^{\frac{N-2}{2}} \left(\prod_{q=1}^p \left(\frac{S - 2q}{S - 2q - 1} \right) \right).$$

We now show that the inequality in (32) holds if the average term of the summation is less than two. Dividing both sides by $\frac{N-2}{2}$ produces

$$\begin{aligned} \frac{2}{N-2}(N-2) &> \frac{2}{N-2} \sum_{p=1}^{\frac{N-2}{2}} \left(\prod_{q=1}^p \left(\frac{S - 2q}{S - 2q - 1} \right) \right) \\ 2 &> \frac{2}{N-2} \sum_{p=1}^{\frac{N-2}{2}} \left(\prod_{q=1}^p \left(\frac{S - 2q}{S - 2q - 1} \right) \right). \end{aligned}$$

The term on the right side of the inequality is an average since there are $\frac{N-2}{2}$ terms in the summation.

Let $T_S^p = \prod_{q=1}^p \left(\frac{S-2q}{S-2q-1} \right)$ and define $A_{N,S} = \frac{2}{N-2} \sum_{p=1}^{\frac{N-2}{2}} T_S^p$ as the average of the T_S^p for $p = 1, 2, \dots, \frac{N-2}{2}$.

We know that $T_S^{p+1} > T_S^p$ for $p \geq 1$ since $T_S^{p+1} = (T_S^p) \left(\frac{S-2p}{S-2p-1} \right)$ and $\frac{S-2p}{S-2p-1} > 1$. Therefore, T_S^p is increasing in p .

An increase in the number of periods, N , for any given S will increase the average as shown in the following:

$$\begin{aligned} A_{N+2,S} &= \frac{2}{N} \left(\sum_{p=1}^{\frac{N}{2}} T_S^p \right) \\ &= \frac{2}{N} \left(\sum_{p=1}^{\frac{N-2}{2}} T_S^p + T_S^{\frac{N}{2}} \right) \\ &= \frac{2}{N} \left(\left(\frac{N-2}{2} \right) A_{N,S} + T_S^{\frac{N}{2}} \right). \end{aligned}$$

Because T_S^p is increasing in p , $T_S^{\frac{N}{2}}$ is greater than each T_S^p included in the average $A_{N,S}$. Therefore, $T_S^{\frac{N}{2}}$ must also be greater than the average value of T_S^p and

$$\frac{2}{N} \left(\left(\frac{N-2}{2} \right) A_{N,S} + T_S^{\frac{N}{2}} \right) > \frac{2}{N} \left(\left(\frac{N-2}{2} \right) A_{N,S} + A_{N,S} \right).$$

Simplifying the right side of the inequality gives

$$\frac{2}{N} \left(\left(\frac{N-2}{2} \right) A_{N,S} + A_{N,S} \right) = \frac{2}{N} \left(A_{N,S} \left(1 + \frac{N}{2} - 1 \right) \right) = A_{N,S}.$$

Therefore, given any S , the average is increasing with an increase in N and τ will be greatest (because $A_{N,S}$ will be largest) when N is as close to S as possible. If S is even, this will be when $N = S - 2$; if S is odd, it will be when $N = S - 1$. We now show that as N increases, $A_{N,S}$ continues to remain less than two. We consider the two subcases when S is even or odd.

Subcase 1: S is odd. In this case, the lowest value of S that satisfies the assumption that $S \geq N + 1$ when $N \geq 4$ is $S = 5$ leaving the only possible value of N as $N = 4$.

$$\begin{aligned} A_{4,5} &= \frac{2}{N-2} \sum_{p=1}^1 \left[\prod_{q=1}^1 \left(\frac{S-2q}{S-2q-1} \right) \right] \\ &= \frac{S-2}{S-3} = \frac{3}{2} < 2. \end{aligned}$$

In this case $\frac{NS-S-N}{S(S-1)} > \tau$ and $1 - \eta > 0$. Therefore, we can conclude that the supplier will prefer a forward auction if and only if $B > \frac{1+\eta}{1-\eta}$.

Now, suppose that $A_{N,N+1} < 2$ for some even $N \geq 4$.

$$\begin{aligned} A_{N,N+1} &= \frac{2}{N-2} \left[\sum_{p=1}^{\frac{N-2}{2}} T_S^p \right] \\ &= \frac{2}{N-2} \left[\sum_{p=1}^{\frac{N-2}{2}} \left(\prod_{q=1}^p \left(\frac{S-2q}{S-2q-1} \right) \right) \right]. \end{aligned}$$

We increase N by two and simplify as follows:

$$\begin{aligned} A_{N+2,N+3} &= \frac{2}{N} \left[\sum_{p=1}^{\frac{N}{2}} T_{S+2}^p \right] \\ &= \frac{2}{N} \left[\sum_{p=1}^{\frac{N}{2}} \left(\prod_{q=1}^p \left(\frac{(S+2)-2q}{(S+2)-2q-1} \right) \right) \right] \\ &= \frac{2}{N} \left[\left(\frac{S}{S-1} \right) + \left(\frac{S}{S-1} \right) \left(\frac{S-2}{S-3} \right) \right. \\ &\quad \left. + \dots + \left(\frac{S}{S-1} \right) \left(\frac{S-2}{S-3} \right) \dots \left(\frac{S-N+2}{S-N+1} \right) \right]. \end{aligned}$$

We factor the common term $\frac{S}{S-1}$ as follows:

$$\begin{aligned} &\frac{2}{N} \left[\left(\frac{S}{S-1} \right) + \left(\frac{S}{S-1} \right) \left(\frac{S-2}{S-3} \right) + \dots + \left(\frac{S}{S-1} \right) \left(\frac{S-2}{S-3} \right) \dots \left(\frac{S-N+2}{S-N+1} \right) \right] \\ &= \frac{2}{N} \left[\frac{S}{S-1} \left[1 + \left(\frac{S-2}{S-3} \right) + \left(\frac{S-2}{S-3} \right) \left(\frac{S-4}{S-5} \right) + \dots + \left(\frac{S-2}{S-3} \right) \left(\frac{S-4}{S-5} \right) \dots \left(\frac{S-N+2}{S-N+1} \right) \right] \right]. \end{aligned}$$

We substitute $S = N + 1$ and condense the product terms to obtain

$$\begin{aligned}
& \frac{2}{N} \left[\frac{S}{S-1} \left[1 + \left(\frac{S-2}{S-3} \right) + \left(\frac{S-2}{S-3} \right) \left(\frac{S-4}{S-5} \right) + \dots + \left(\frac{S-2}{S-3} \right) \left(\frac{S-4}{S-5} \right) \dots \left(\frac{S-N+2}{S-N+1} \right) \right] \right] \\
&= \frac{2}{N} \left[\frac{S}{S-1} \left[1 + \sum_{p=1}^{\frac{N-2}{2}} \left(\prod_{q=1}^p \left(\frac{S-2q}{S-2q+1} \right) \right) \right] \right] \\
&= \frac{2}{N} \left[\frac{N+1}{N} \left[1 + \sum_{p=1}^{\frac{N-2}{2}} \left(\prod_{q=1}^p \left(\frac{S-2q}{S-2q+1} \right) \right) \right] \right] \\
&= \frac{2}{N} \left[\frac{N+1}{N} \left[1 + \frac{N-2}{2} A_{N,N+1} \right] \right] \\
&= \frac{2N+2}{N^2} + \frac{N^2-N-2}{N^2} A_{N,N+1} \\
&< \frac{2N+2}{N^2} + \frac{2N^2-2N-4}{N^2} \\
&= \frac{2N^2-2}{N^2} = 2 - \frac{2}{N^2} < 2.
\end{aligned}$$

Therefore, $A_{N,N+1} < 2$ for all even $N \geq 4$. Since $S = N + 1$ in this case, we can equivalently say $A_{S-1,S} < 2$ for all odd $S \geq 5$. We conclude that in this case $\frac{NS-S-N}{S(S-1)} > \tau$ and $1 - \eta > 0$. Therefore, the supplier will prefer a forward auction if and only if $B > \frac{1+\eta}{1-\eta}$.

We now prove by induction the case when S is even.

Subcase 2: S is even. In this case, the lowest value of S that satisfies the assumption that $S \geq N + 1$ when $N \geq 4$ is $S = 6$ leaving the only possible value of N as $N = 4$.

$$\begin{aligned}
A_{4,6} &= \frac{2}{N-2} \sum_{p=1}^1 \left[\prod_{q=1}^1 \left(\frac{S-2q}{S-2q-1} \right) \right] \\
&= \frac{S-2}{S-3} = \frac{4}{3} < 2.
\end{aligned}$$

In this case $\frac{NS-S-N}{S(S-1)} > \tau$ and $1 - \eta > 0$. Therefore, we can conclude that the supplier will prefer a forward auction if and only if $B > \frac{1+\eta}{1-\eta}$.

Now, suppose that $A_{N,N+2} < 2$ for some even $N \geq 4$.

$$\begin{aligned}
A_{N,N+2} &= \frac{2}{N-2} \left[\sum_{p=1}^{\frac{N-2}{2}} T_S^p \right] \\
&= \frac{2}{N-2} \left[\sum_{p=1}^{\frac{N-2}{2}} \left(\prod_{q=1}^p \left(\frac{S-2q}{S-2q-1} \right) \right) \right].
\end{aligned}$$

We increase N by two and simplify as follows:

$$\begin{aligned}
A_{N+2,N+4} &= \frac{2}{N} \left[\sum_{p=1}^{\frac{N}{2}} T_{S+2}^p \right] \\
&= \frac{2}{N} \left[\sum_{p=1}^{\frac{N}{2}} \left(\prod_{q=1}^p \left(\frac{(S+2)-2q}{(S+2)-2q-1} \right) \right) \right] \\
&= \frac{2}{N} \left[\left(\frac{S}{S-1} \right) + \left(\frac{S}{S-1} \right) \left(\frac{S-2}{S-3} \right) \right. \\
&\quad \left. + \dots + \left(\frac{S}{S-1} \right) \left(\frac{S-2}{S-3} \right) \dots \left(\frac{S-N+2}{S-N+1} \right) \right].
\end{aligned}$$

We factor the common term $\frac{S}{S-1}$ as follows:

$$\begin{aligned}
&\frac{2}{N} \left[\left(\frac{S}{S-1} \right) + \left(\frac{S}{S-1} \right) \left(\frac{S-2}{S-3} \right) + \dots + \left(\frac{S}{S-1} \right) \left(\frac{S-2}{S-3} \right) \dots \left(\frac{S-N+2}{S-N+1} \right) \right] \\
&= \frac{2}{N} \left[\frac{S}{S-1} \left[1 + \left(\frac{S-2}{S-3} \right) + \left(\frac{S-2}{S-3} \right) \left(\frac{S-4}{S-5} \right) + \dots + \left(\frac{S-2}{S-3} \right) \left(\frac{S-4}{S-5} \right) \dots \left(\frac{S-N+2}{S-N+1} \right) \right] \right].
\end{aligned}$$

We substitute $S = N + 2$ and condense the product terms to obtain

$$\begin{aligned}
&\frac{2}{N} \left[\frac{S}{S-1} \left[1 + \left(\frac{S-2}{S-3} \right) + \left(\frac{S-2}{S-3} \right) \left(\frac{S-4}{S-5} \right) + \dots + \left(\frac{S-2}{S-3} \right) \left(\frac{S-4}{S-5} \right) \dots \left(\frac{S-N+2}{S-N+1} \right) \right] \right] \\
&= \frac{2}{N} \left[\frac{S}{S-1} \left[1 + \sum_{p=1}^{\frac{N-2}{2}} \left(\prod_{q=1}^p \left(\frac{S-2q}{S-2q+1} \right) \right) \right] \right] \\
&= \frac{2}{N} \left[\frac{N+2}{N+1} \left[1 + \sum_{p=1}^{\frac{N-2}{2}} \left(\prod_{q=1}^p \left(\frac{S-2q}{S-2q+1} \right) \right) \right] \right] \\
&= \frac{2}{N} \left[\frac{N+2}{N+1} \left[1 + \frac{N-2}{2} A_{N,N+2} \right] \right] \\
&= \frac{2N+4}{N^2+N} + \frac{N^2-4}{N^2+N} A_{N,N+2} \\
&< \frac{2N+4}{N^2+N} + \frac{2N^2-8}{N^2+N} \\
&= \frac{2N^2+2N-4}{N^2+N} = 2 - \frac{4}{N^2+N} < 2.
\end{aligned}$$

Therefore, $A_{N,N+2} < 2$ for all even $N \geq 4$. Since $S = N+2$ in this case, we can equivalently say $A_{S-2,S} < 2$ for all even $S \geq 6$. We conclude that in this case $\frac{NS-S-N}{S(S-1)} > \tau$ and $1-\eta > 0$.

Therefore, the supplier will prefer a forward auction if and only if $B > \frac{1+\eta}{1-\eta}$.

Case 3: N is odd and $N \geq 3$. When N is odd and $N \geq 3$, we determine if the inequality $\frac{NS-S-N}{S(S-1)} > \tau$ is valid for any S when $N \geq 3$. In this case, $\frac{NS-S-N}{S(S-1)} > \tau$ becomes

$$\frac{NS-S-N}{S(S-1)} > \sum_{p=0}^{\frac{N-3}{2}} \left(\prod_{q=0}^p \left(\frac{S-2q-1}{S-2q} \right) \right) \frac{1}{S-2p-2}. \quad (35)$$

We begin by showing this inequality to hold for $N = 3$ and $S = 4$ and $N = 3$ and $S = 6$ then continue with the proof for $N \geq 5$ and $S \geq 6$. When $N = 3$ and $S = 4$, the inequality

in (35) becomes

$$\begin{aligned}\frac{NS - S - N}{S(S-1)} &> \sum_{p=0}^0 \left(\prod_{q=0}^0 \left(\frac{S-2q-1}{S-2q} \right) \right) \frac{1}{S-2p-2}, \\ \frac{5}{12} &> \left(\frac{S-1}{S} \right) \frac{1}{S-2} \\ \frac{5}{12} &> \frac{3}{8}.\end{aligned}$$

When $N = 3$ and $S = 5$, the inequality in (35) becomes

$$\begin{aligned}\frac{NS - S - N}{S(S-1)} &> \sum_{p=0}^0 \left(\prod_{q=0}^0 \left(1 - \frac{1}{S-2q} \right) \right) \frac{1}{S-2p-2}, \\ \frac{7}{20} &> \left(\frac{S-1}{S} \right) \frac{1}{S-2} \\ \frac{7}{20} &> \frac{4}{15}.\end{aligned}$$

We now continue with the proof for $N \geq 5$ and $S \geq 6$. The left side of the inequality in (35) can be expressed as

$$\begin{aligned}\frac{NS - S - N}{S(S-1)} &= \frac{N-1}{S-1} - \frac{N}{S(S-1)} \\ &= \frac{1}{S} \left(N - \frac{S}{S-1} \right).\end{aligned}\tag{36}$$

The right side of the inequality in (35) can be expanded as follows:

$$\begin{aligned}\sum_{p=0}^{\frac{N-3}{2}} \left(\prod_{q=0}^p \left(\frac{S-2q-1}{S-2q} \right) \right) \frac{1}{S-2p-2} &= \left(\frac{S-1}{S} \right) \left(\frac{1}{S-2} \right) + \left(\frac{S-1}{S} \right) \left(\frac{S-3}{S-2} \right) \left(\frac{1}{S-4} \right) \\ &+ \dots + \left(\frac{S-1}{S} \right) \left(\frac{S-3}{S-2} \right) \dots \left(\frac{S-N+2}{S-N+3} \right) \left(\frac{1}{S-N+1} \right).\end{aligned}$$

We regroup the terms to obtain

$$\begin{aligned}\sum_{p=0}^{\frac{N-3}{2}} \left(\prod_{q=0}^p \left(\frac{S-2q-1}{S-2q} \right) \right) \frac{1}{S-2p-2} &= \frac{1}{S} \left[\left(\frac{S-1}{S-2} \right) + \left(\frac{S-1}{S-2} \right) \left(\frac{S-3}{S-4} \right) \right. \\ &\left. + \dots + \left(\frac{S-1}{S-2} \right) \left(\frac{S-3}{S-4} \right) \dots \left(\frac{S-N+2}{S-N+1} \right) \right].\end{aligned}$$

The product terms within the brackets can be expressed as

$$\begin{aligned}&\frac{1}{S} \left[\left(\frac{S-1}{S-2} \right) + \left(\frac{S-3}{S-4} \right) \left(\frac{S-5}{S-6} \right) + \dots + \left(\frac{S-1}{S-2} \right) \left(\frac{S-3}{S-4} \right) \dots \left(\frac{S-N+2}{S-N+1} \right) \right] \\ &= \frac{1}{S} \sum_{p=0}^{\frac{N-3}{2}} \left(\prod_{q=0}^p \left(\frac{S-2q-1}{S-2q-2} \right) \right).\end{aligned}$$

Therefore,

$$\tau = \frac{1}{S} \sum_{p=0}^{\frac{N-3}{2}} \left(\prod_{q=0}^p \left(\frac{S-2q-1}{S-2q-2} \right) \right).\tag{37}$$

Let $R_S^p = \prod_{q=0}^p (\frac{S-2q-1}{S-2q-2})$ and define $M_{N,S} = \frac{2}{N-1} \sum_{p=0}^{\frac{N-3}{2}} R_S^p$ as the average of the R_S^p for $p = 0, 1, 2, \dots, \frac{N-3}{2}$.

We know that $R_S^{p+1} > R_S^p$ for $p \geq 0$ since $R_S^{p+1} = (R_S^p)(\frac{S-2p-1}{S-2p-2})$ and $\frac{S-2p-1}{S-2p-2} > 1$. Therefore, R_S^p is increasing in p .

An increase in the number of periods, N , for any given S will increase $M_{N,S}$ as shown in the following:

$$\begin{aligned} M_{N+2,S} &= \frac{2}{(N+1)} \left[\sum_{p=0}^{\frac{N-1}{2}} R_S^p \right] \\ &= \frac{2}{(N+1)} \left[\sum_{p=0}^{\frac{N-3}{2}} R_S^p + R_S^{\frac{N-1}{2}} \right] \\ &= \frac{2}{(N+1)} \left[\left(\frac{N-1}{2} \right) M_{N,S} + R_S^{\frac{N-1}{2}} \right]. \end{aligned}$$

Because R_S^p is increasing in p , $R_S^{\frac{N-1}{2}}$ is greater than each R_S^p included in the average $M_{N,S}$. Therefore, $R_S^{\frac{N-1}{2}}$ is also greater than the average of R_S^p and

$$\frac{2}{N+1} \left[\left(\frac{N-1}{2} \right) M_{N,S} + R_S^{\frac{N-1}{2}} \right] > \frac{2}{N+1} \left[\left(\frac{N-1}{2} \right) M_{N,S} + M_{N,S} \right].$$

Simplifying the right side of the inequality gives

$$\begin{aligned} \frac{2}{N+1} \left[\left(\frac{N-1}{2} \right) M_{N,S} + M_{N,S} \right] &= \frac{2}{N+1} \left[M_{N,S} \left(1 + \frac{N-1}{2} \right) \right] \\ &= \frac{2}{N+1} \left[M_{N,S} \left(\frac{N+1}{2} \right) \right] \\ &= M_{N,S}. \end{aligned}$$

Therefore, given any S , $M_{N,S}$ is increasing with an increase in N and τ will be greatest (because $M_{N,S}$ will be largest) when N is as close to S as possible. If S is even, this will be when $N = S - 1$; if S is odd, it will be when $N = S - 2$. We now prove by induction that $M_{S-1,S} < \frac{2(S+1)}{S+2}$ for all $S \geq 6$. We consider the two subcases when S is even or odd.

Subcase 1: S is odd. In this case, $S = N + 2$. We begin with $S = 7$.

$$\begin{aligned} M_{5,7} &= \left(\frac{2}{N-1} \right) \sum_{p=0}^1 \left[\prod_{q=0}^p \left(\frac{S-2q-1}{S-2q-2} \right) \right] \\ &= \left(\frac{1}{3} \right) \left[\frac{5}{4} + \left(\frac{5}{4} \right) \left(\frac{3}{2} \right) \right] \\ &= \frac{25}{24} < \frac{2(S+1)}{S+2} = \frac{14}{8}. \end{aligned}$$

Now, suppose that $M_{S-2,S} < \frac{2(S+1)}{S+2}$ for some odd $S \geq 7$.

$$\begin{aligned} M_{S,S+2} &= \frac{2}{(N-1)+2} \sum_{p=0}^{\frac{(N-3)+2}{2}} R_{S+2}^p \\ &= \frac{2}{N+1} \sum_{p=0}^{\frac{(N-3)+2}{2}} R_{S+2}^p. \end{aligned}$$

We simplify this expression as follows:

$$\begin{aligned} M_{S,S+2} &= \left(\frac{2}{N+1}\right) \left[\sum_{p=0}^{\frac{N-1}{2}} R_{S+2}^p \right] \\ &= \left(\frac{2}{N+1}\right) \left[\sum_{p=0}^{\frac{N-1}{2}} \left(\prod_{q=0}^p \left(\frac{(S+2)-2q-1}{(S+2)-2q} \right) \right) \right] \\ &= \left(\frac{2}{N+1}\right) \left[\left(\frac{S+1}{S} \right) + \left(\frac{S+1}{S} \right) \left(\frac{S-1}{S-2} \right) \right. \\ &\quad \left. + \dots + \left(\frac{S+1}{S} \right) \left(\frac{S-1}{S-2} \right) \dots \left(\frac{S-N+2}{S-N+1} \right) \right]. \end{aligned}$$

We factor the common term $\frac{S+1}{S}$ as follows:

$$\begin{aligned} &= \left(\frac{2}{N+1}\right) \left[\left(\frac{S+1}{S} \right) + \left(\frac{S+1}{S} \right) \left(\frac{S-1}{S-2} \right) + \dots + \left(\frac{S+1}{S} \right) \left(\frac{S-1}{S-2} \right) \dots \left(\frac{S-N+2}{S-N+1} \right) \right] \\ &= \left(\frac{2}{N+1}\right) \left[\frac{S+1}{S} \left[1 + \left(\frac{S-1}{S-2} \right) + \left(\frac{S-1}{S-2} \right) \left(\frac{S-3}{S-4} \right) \right. \right. \\ &\quad \left. \left. + \dots + \left(\frac{S-1}{S-2} \right) \left(\frac{S-3}{S-4} \right) \dots \left(\frac{S-N+2}{S-N+1} \right) \right] \right]. \end{aligned}$$

We condense the product terms to obtain

$$\begin{aligned} &\left(\frac{2}{N+1}\right) \left[\frac{S+1}{S} \left[1 + \left(\frac{S-1}{S-2} \right) + \left(\frac{S-1}{S-2} \right) \left(\frac{S-3}{S-4} \right) \right. \right. \\ &\quad \left. \left. + \dots + \left(\frac{S-1}{S-2} \right) \left(\frac{S-3}{S-4} \right) \dots \left(\frac{S-N+2}{S-N+1} \right) \right] \right] \\ &= \left(\frac{2}{N+1}\right) \left[\frac{S+1}{S} \left[1 + \sum_{p=0}^{\frac{N-3}{2}} \left(\prod_{q=0}^p \left(\frac{S-2q-1}{S-2q+2} \right) \right) \right] \right] \\ &= \left(\frac{2}{N+1}\right) \left[\frac{S+1}{S} \left[1 + \left(\frac{N-1}{2} \right) M_{S-2,S} \right] \right] \end{aligned}$$

This can be expressed as follows:

$$\begin{aligned}
&= \left(\frac{2}{N+1}\right) \left[\frac{S+1}{S} \left[1 + \left(\frac{N-1}{2}\right) M_{S-2,S} \right] \right] \\
&< \left(\frac{2}{N+1}\right) \left[\frac{S+1}{S} \left[1 + \frac{N-1}{2} \right] \frac{2(S+1)}{S+2} \right] \\
&= \left(\frac{2}{N+1}\right) \left[\frac{S+1}{S} \left[\frac{S+2+NS-S+N-1}{S+2} \right] \right] \\
&= \left[\frac{2(S+1)}{S+2} \right] \left[\frac{S+2+NS-S+N-1}{S(N+1)} \right] \\
&= \left[\frac{2(S+1)}{S+2} \right] \left[\frac{NS+N+1}{S(N+1)} \right] \\
&= \left[\frac{2(S+1)}{S+2} \right] \left[\left(\frac{NS}{S(N+1)} + \frac{N+1}{S(N+1)} \right) \right] \\
&= \left[\frac{2(S+1)}{S+2} \right] \left(\frac{N}{(N+1)} + \frac{1}{S} \right) \\
&< \frac{2(S+1)}{S+2} \left(\frac{N}{(N+1)} + \frac{1}{N+1} \right) \\
&= \frac{2(S+1)}{S+2} \\
&< \frac{2(S+3)}{S+4}.
\end{aligned}$$

Therefore, we can use this result, (37), and the previous definition $M_{N,S} = \frac{2}{N-1} \sum_{p=0}^{\frac{N-3}{2}} R_S^p$, to show that $\tau < \frac{(N-1)(S+1)}{S(S+2)}$ as follows:

$$\begin{aligned}
\left(\frac{2}{N-1}\right) \sum_{p=0}^{\frac{N-3}{2}} R_S^p &< \frac{2(S+1)}{S+2} \\
\sum_{p=0}^{\frac{N-3}{2}} R_S^p &< \left(\frac{N-1}{2}\right) \left(\frac{2(S+1)}{S+2}\right) \\
\frac{1}{S} \sum_{p=0}^{\frac{N-3}{2}} R_S^p &< \frac{1}{S} \left(\frac{N-1}{2}\right) \left(\frac{2(S+1)}{S+2}\right) \\
\frac{1}{S} \sum_{p=0}^{\frac{N-3}{2}} \left(\prod_{q=0}^p \left(\frac{S-2q-1}{S-2q-2} \right) \right) &< \frac{1}{S} \left(\frac{N-1}{2}\right) \left(\frac{2(S+1)}{S+2}\right) \\
\tau &< \left(\frac{1}{S}\right) \left(\frac{N-1}{2}\right) \left(\frac{2(S+1)}{S+2}\right) \\
\tau &< \frac{(N-1)(S+1)}{S(S+2)}.
\end{aligned}$$

Now we prove that $\tau < \frac{NS-N-S}{S(S-1)}$ by showing

$$\tau < \frac{(N-1)(S+1)}{S(S+2)} < \frac{NS-N-S}{S(S-1)}.$$

We show that $\frac{(N-1)(S+1)}{S(S+2)} < \frac{NS-N-S}{S(S-1)}$ noting that $NS-N-S = (N-1)(S-1) - 1$.

So,

$$\begin{aligned}
& \frac{(N-1)(S+1)}{S(S+2)} < \frac{NS-N-S}{S(S-1)} \\
\Leftrightarrow & \frac{(N-1)(S+1)}{S(S+2)} < \frac{(N-1)(S-1)-1}{S(S-1)} \\
\Leftrightarrow & \frac{(N-1)(S+1)}{S+2} < \frac{(N-1)(S-1)-1}{S-1} \\
\Leftrightarrow & (N-1)(S+1)(S-1) < (N-1)(S-1)(S+2) - (S+2) \\
\Leftrightarrow & 0 < (N-1)(S-1)((S+2) - (S+1)) - (S+2) \\
\Leftrightarrow & S+2 < (N-1)(S-1) \\
\Leftrightarrow & \frac{S+2}{S-1} < N-1.
\end{aligned}$$

Since $\frac{S+2}{S-1} < 2$ for all $S \geq 7$ and since $N \geq 5$ in this case, this inequality holds. We conclude that in this case $\frac{NS-S-N}{S(S-1)} > \tau$ and $1-\eta > 0$. Therefore, for $N \geq 5$, the inequality in (35) holds. Therefore, the supplier will prefer a forward auction if and only if $B > \frac{1+\eta}{1-\eta}$.

We now prove by induction that $M_{S-1,S} < \frac{2(S+1)}{S+2}$ for all $S \geq 6$ when S is even.

Subcase 2: S is even. In this case, $S = N+1$. We begin with $S = 6$. The corresponding value of N is $N = 5$.

$$\begin{aligned}
M_{5,6} &= \left(\frac{2}{N-1}\right) \sum_{p=0}^1 \left[\prod_{q=0}^1 \left(\frac{S-2q-1}{S-2q-2}\right) \right] \\
&= \left(\frac{1}{2}\right) \left[\frac{5}{4} + \left(\frac{5}{4}\right) \left(\frac{3}{2}\right) \right] \\
&= \frac{25}{16} < \frac{2(S+1)}{S+2} = \frac{14}{8}.
\end{aligned}$$

Now, suppose that $M_{S-1,S} < \frac{2(S+1)}{S+2}$ for some even $S \geq 6$.

$$\begin{aligned}
M_{S+1,S+2} &= \left(\frac{2}{(N-1)+2}\right) \sum_{p=0}^{\frac{(N-3)+2}{2}} R_{S+2}^p \\
&= \left(\frac{2}{N+1}\right) \sum_{p=0}^{\frac{(N-3)+2}{2}} R_{S+2}^p.
\end{aligned}$$

We simplify this expression as follows:

$$\begin{aligned}
M_{S+1,S+2} &= \left(\frac{2}{N+1}\right) \left[\sum_{p=0}^{\frac{N-1}{2}} R_{S+2}^p \right] \\
&= \left(\frac{2}{N+1}\right) \left[\sum_{p=0}^{\frac{N-1}{2}} \left(\prod_{q=0}^p \left(\frac{(S+2)-2q-1}{(S+2)-2q} \right) \right) \right] \\
&= \left(\frac{2}{N+1}\right) \left[\left(\frac{S+1}{S} \right) + \left(\frac{S+1}{S} \right) \left(\frac{S-1}{S-2} \right) \right. \\
&\quad \left. + \dots + \left(\frac{S+1}{S} \right) \left(\frac{S-1}{S-2} \right) \dots \left(\frac{S-N+2}{S-N+1} \right) \right].
\end{aligned}$$

We factor the common term $\frac{S+1}{S}$ as follows:

$$\begin{aligned}
&= \left(\frac{2}{N+1}\right) \left[\left(\frac{S+1}{S} \right) + \left(\frac{S+1}{S} \right) \left(\frac{S-1}{S-2} \right) + \dots + \left(\frac{S+1}{S} \right) \left(\frac{S-1}{S-2} \right) \dots \left(\frac{S-N+2}{S-N+1} \right) \right] \\
&= \left(\frac{2}{N+1}\right) \left[\frac{S+1}{S} \left[1 + \left(\frac{S-1}{S-2} \right) + \left(\frac{S-1}{S-2} \right) \left(\frac{S-3}{S-4} \right) \right. \right. \\
&\quad \left. \left. + \dots + \left(\frac{S-1}{S-2} \right) \left(\frac{S-3}{S-4} \right) \dots \left(\frac{S-N+2}{S-N+1} \right) \right] \right].
\end{aligned}$$

We condense the product terms to obtain

$$\begin{aligned}
&\left(\frac{2}{N+1}\right) \left[\frac{S+1}{S} \left[1 + \left(\frac{S-1}{S-2} \right) + \left(\frac{S-1}{S-2} \right) \left(\frac{S-3}{S-4} \right) \right. \right. \\
&\quad \left. \left. + \dots + \left(\frac{S-1}{S-2} \right) \left(\frac{S-3}{S-4} \right) \dots \left(\frac{S-N+2}{S-N+1} \right) \right] \right] \\
&= \left(\frac{2}{N+1}\right) \left[\frac{S+1}{S} \left[1 + \sum_{p=0}^{\frac{N-3}{2}} \left(\prod_{q=0}^p \left(\frac{S-2q-1}{S-2q+2} \right) \right) \right] \right] \\
&= \left(\frac{2}{N+1}\right) \left[\frac{S+1}{S} \left[1 + \left(\frac{N-1}{2} \right) M_{S-1,S} \right] \right]
\end{aligned}$$

This can be expressed as follows:

$$\begin{aligned}
&= \left(\frac{2}{N+1}\right) \left[\frac{S+1}{S} \left[1 + \left(\frac{N-1}{2}\right) M_{S-1,S} \right] \right] \\
&< \left(\frac{2}{N+1}\right) \left[\frac{S+1}{S} \left[1 + \frac{N-1}{2} \right] \frac{2(S+1)}{S+2} \right] \\
&= \left(\frac{2}{N+1}\right) \left[\frac{S+1}{S} \left[\frac{S+2+NS-S+N-1}{S+2} \right] \right] \\
&= \left[\frac{2(S+1)}{S+2} \right] \left[\frac{S+2+NS-S+N-1}{S(N+1)} \right] \\
&= \left[\frac{2(S+1)}{S+2} \right] \left[\frac{NS+N+1}{S(N+1)} \right] \\
&= \left[\frac{2(S+1)}{S+2} \right] \left[\left(\frac{NS}{S(N+1)} + \frac{N+1}{S(N+1)} \right) \right] \\
&= \left[\frac{2(S+1)}{S+2} \right] \left(\frac{N}{(N+1)} + \frac{1}{S} \right) \\
&\leq \frac{2(S+1)}{S+2} \left(\frac{N}{(N+1)} + \frac{1}{N+1} \right) \\
&= \frac{2(S+1)}{S+2} \\
&< \frac{2(S+3)}{S+4}.
\end{aligned}$$

Therefore, we can use this result, (37), and the previous definition $M_{N,S} = \frac{2}{N-1} \sum_{p=0}^{\frac{N-3}{2}} R_S^p$, to show that $\tau < \frac{(N-1)(S+1)}{S(S+2)}$ as follows:

$$\begin{aligned}
\left(\frac{2}{N-1}\right) \sum_{p=0}^{\frac{N-3}{2}} R_S^p &< \frac{2(S+1)}{S+2} \\
\sum_{p=0}^{\frac{N-3}{2}} R_S^p &< \left(\frac{N-1}{2}\right) \left(\frac{2(S+1)}{S+2}\right) \\
\frac{1}{S} \sum_{p=0}^{\frac{N-3}{2}} R_S^p &< \frac{1}{S} \left(\frac{N-1}{2}\right) \left(\frac{2(S+1)}{S+2}\right) \\
\frac{1}{S} \sum_{p=0}^{\frac{N-3}{2}} \left(\prod_{q=0}^p \left(\frac{S-2q-1}{S-2q-2} \right) \right) &< \frac{1}{S} \left(\frac{N-1}{2}\right) \left(\frac{2(S+1)}{S+2}\right) \\
\tau &< \left(\frac{1}{S}\right) \left(\frac{N-1}{2}\right) \left(\frac{2(S+1)}{S+2}\right) \\
\tau &< \frac{(N-1)(S+1)}{S(S+2)}.
\end{aligned}$$

Now we prove that $\tau < \frac{NS-N-S}{S(S-1)}$ by showing

$$\tau < \frac{(N-1)(S+1)}{S(S+2)} < \frac{NS-N-S}{S(S-1)}.$$

We show that $\frac{(N-1)(S+1)}{S(S+2)} < \frac{NS-N-S}{S(S-1)}$ noting that $NS-N-S = (N-1)(S-1) - 1$.

So,

$$\begin{aligned}
& \frac{(N-1)(S+1)}{S(S+2)} < \frac{NS-N-S}{S(S-1)} \\
\Leftrightarrow & \frac{(N-1)(S+1)}{S(S+2)} < \frac{(N-1)(S-1)-1}{S(S-1)} \\
\Leftrightarrow & \frac{(N-1)(S+1)}{S+2} < \frac{(N-1)(S-1)-1}{S-1} \\
\Leftrightarrow & (N-1)(S+1)(S-1) < (N-1)(S-1)(S+2) - (S+2) \\
\Leftrightarrow & 0 < (N-1)(S-1)((S+2) - (S+1)) - (S+2) \\
\Leftrightarrow & S+2 < (N-1)(S-1) \\
\Leftrightarrow & \frac{S+2}{S-1} < N-1.
\end{aligned}$$

Since $\frac{S+2}{S-1} < 2$ for all $S \geq 6$ and since $N \geq 5$ in this case, this inequality holds. We conclude that in this case $\frac{NS-S-N}{S(S-1)} > \tau$ and $1-\eta > 0$. Therefore, for $N \geq 5$, the inequality in (35) holds. Therefore, the supplier will prefer a forward auction if and only if $B > \frac{1+\eta}{1-\eta}$.

■

APPENDIX C

EQUILIBRIUM FIGURES AND TABLES

In this appendix, we consider the regions that constitute an equilibrium auction design.

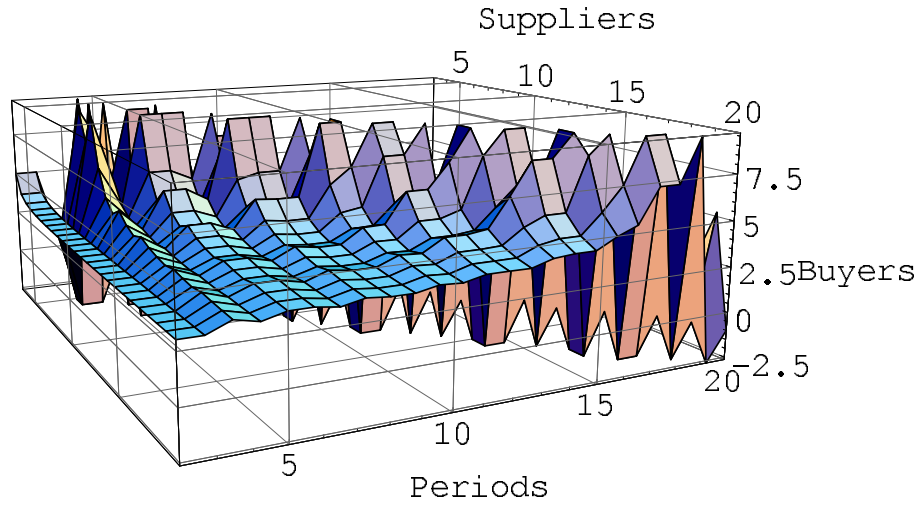


Figure 14: Supplier FA/RFA Indifference Surface with Preference: FA above; RFA below

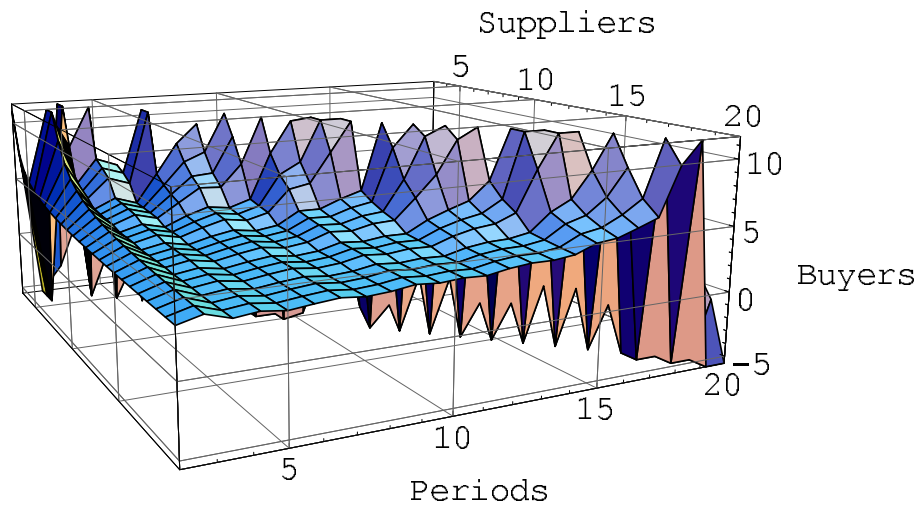


Figure 15: Supplier RA/RFA Indifference Surface with Preference: RA below; RFA above

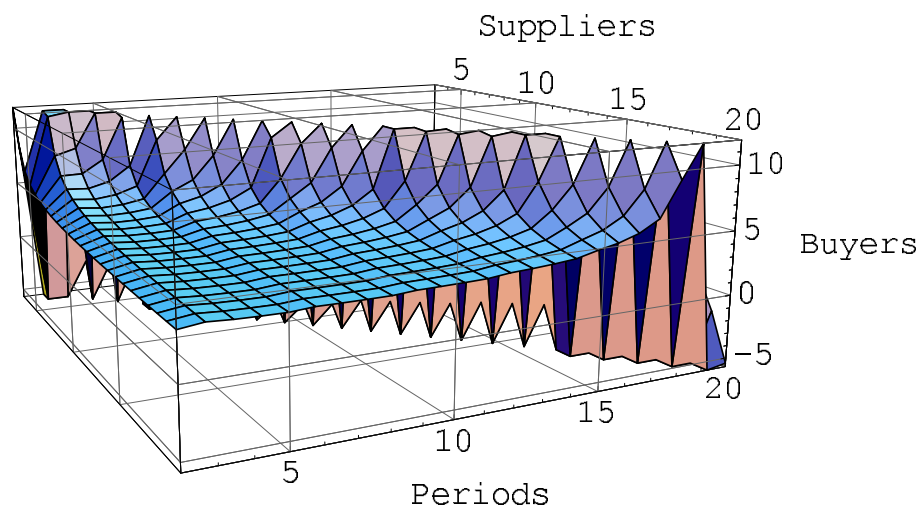


Figure 16: Supplier RA/FA Indifference Surface with Preference: RA below; FA above

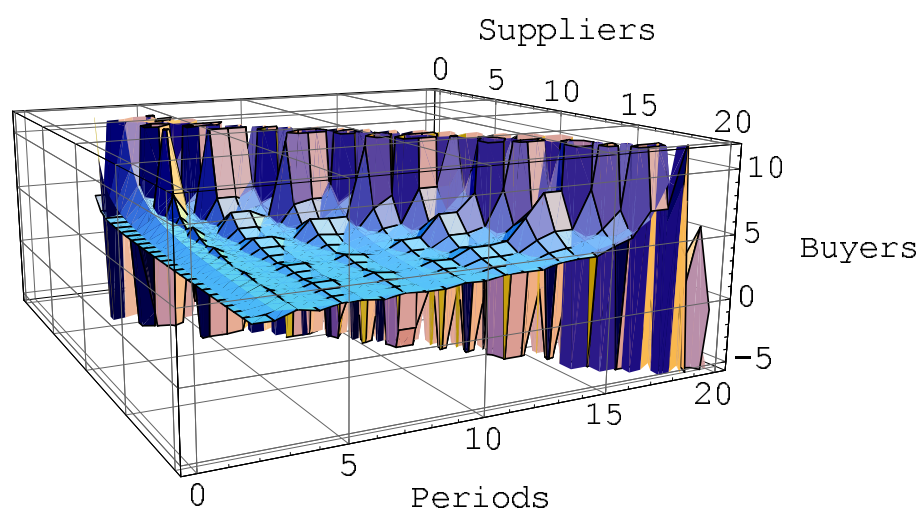


Figure 17: Mesh Supplier FA/RFA and Smooth RA/FA Indifference Surfaces

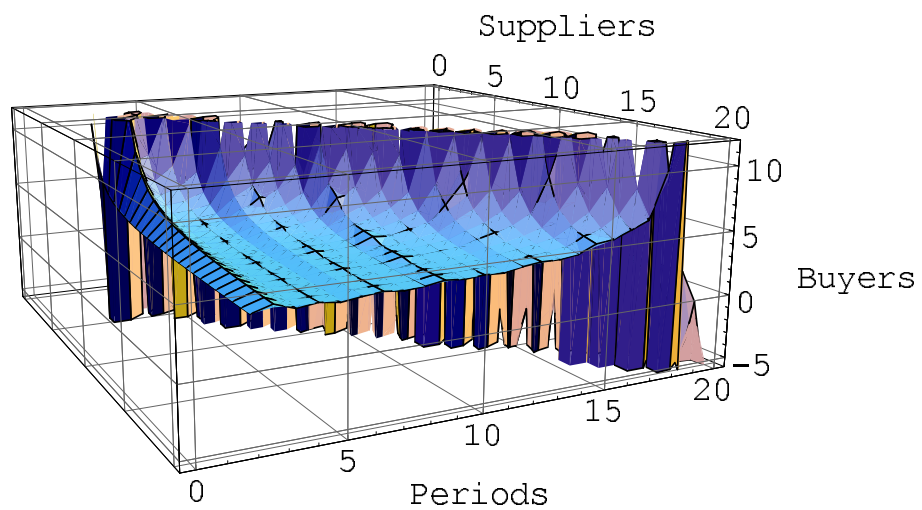


Figure 18: Mesh Supplier RA/RFA and Smooth RA/FA Indifference Surfaces

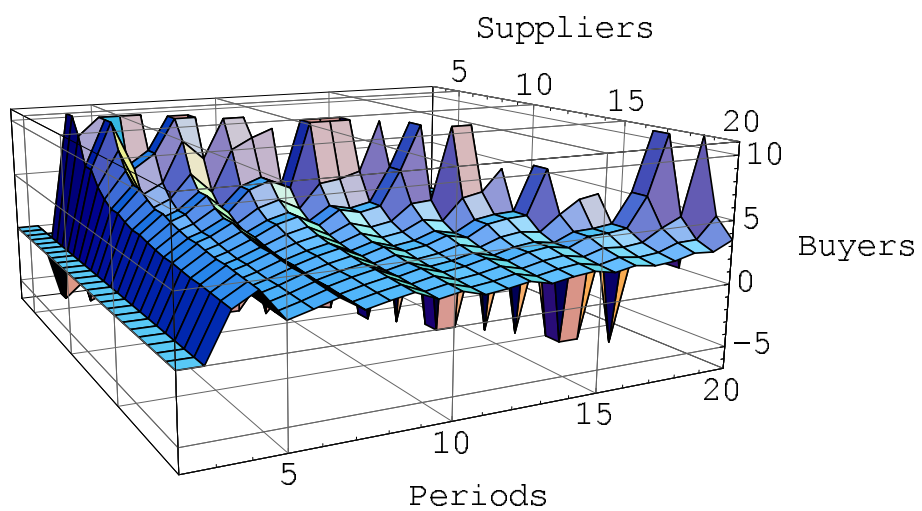


Figure 19: Buyer FA/RFA Indifference Surface with Preference: FA below; RFA above

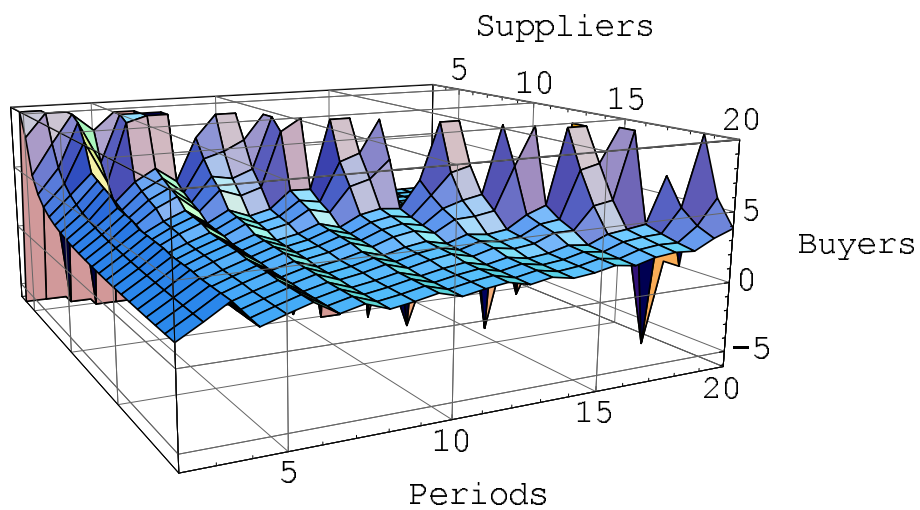


Figure 20: Buyer RA/RFA Indifference Surface with Preference: RA above; RFA below

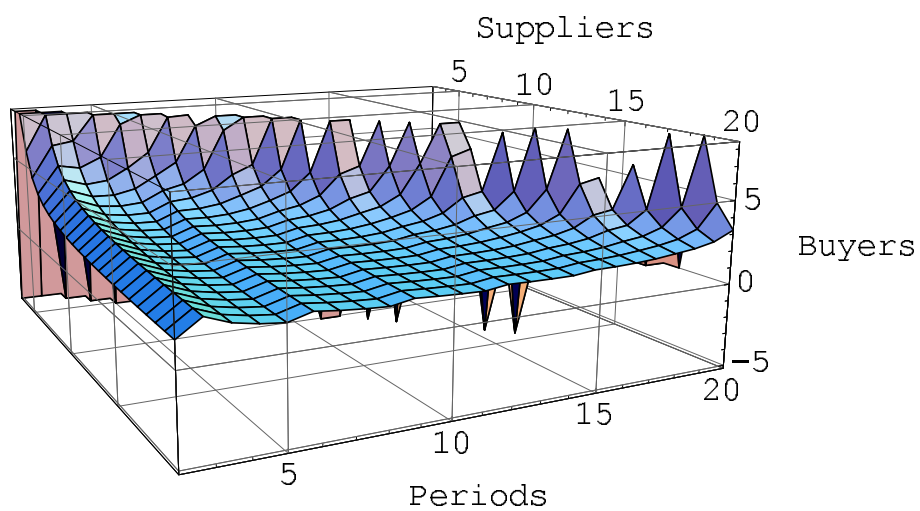


Figure 21: Buyer RA/FA Indifference Surface with Preference: RA above; FA below

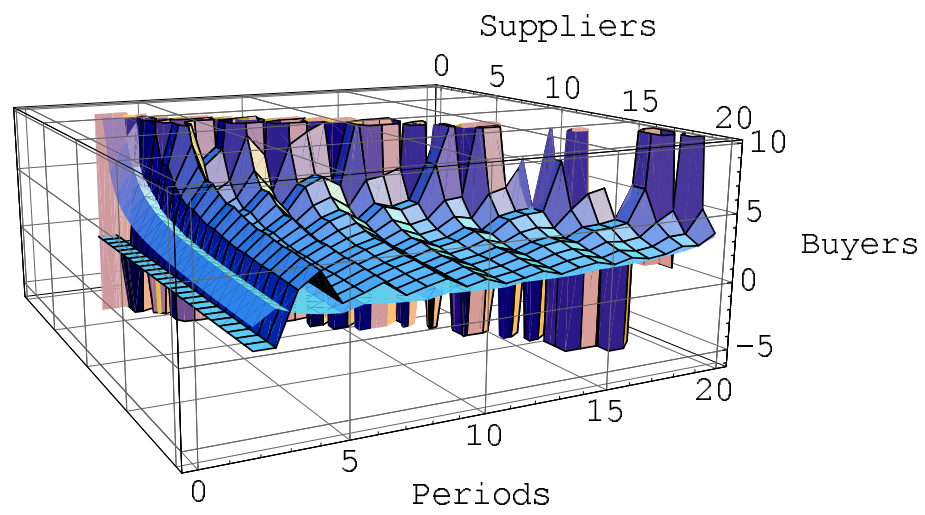


Figure 22: Mesh Buyer FA/RFA and Smooth RA/FA Indifference Surfaces

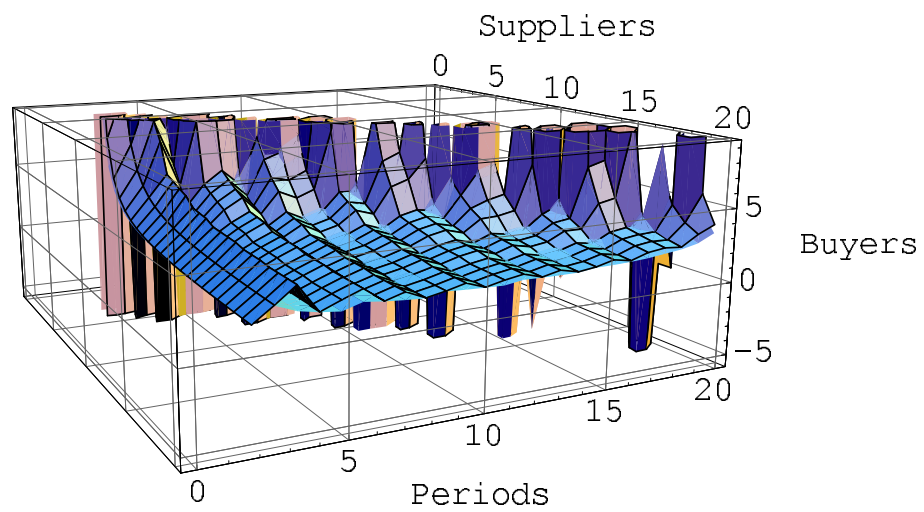


Figure 23: Mesh Buyer RA/RFA and Smooth RA/FA Indifference Surfaces

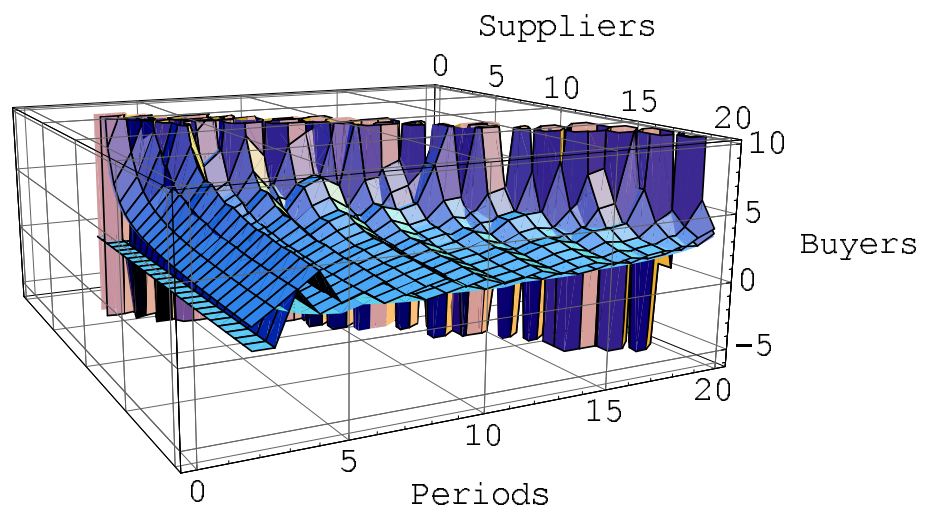


Figure 24: Three Buyer Indifference Surfaces

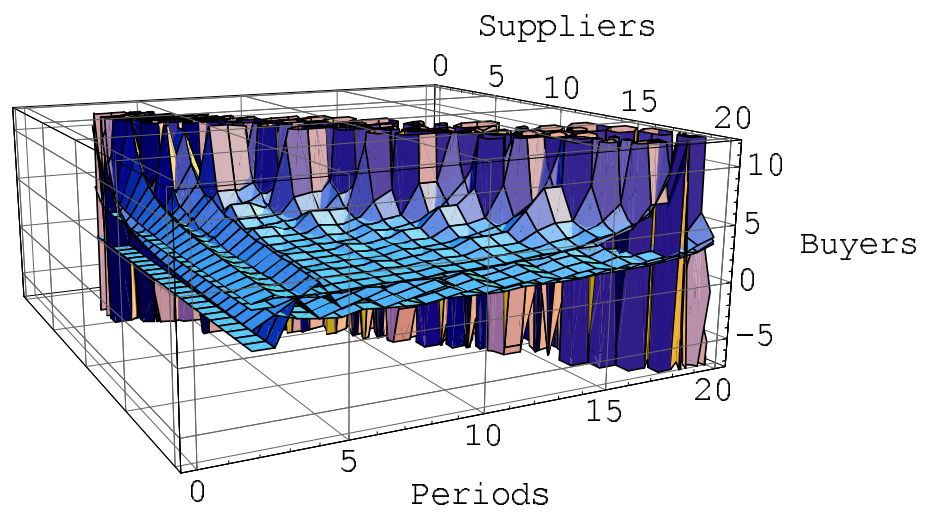


Figure 25: All Six Indifference Surfaces

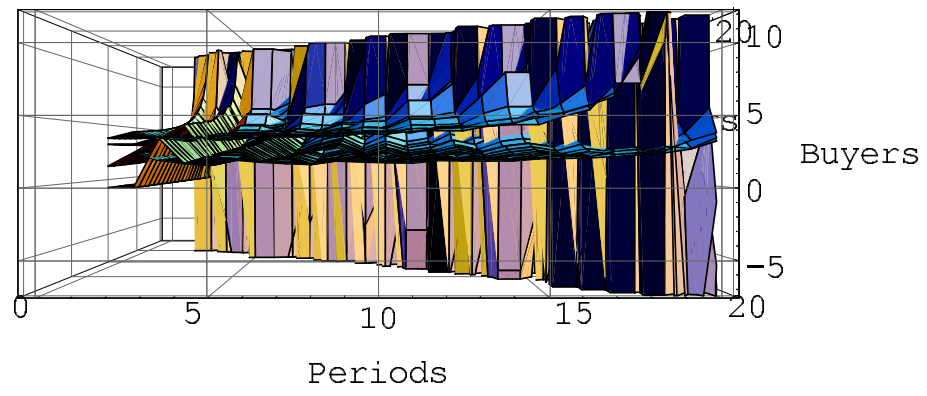


Figure 26: Indifference Surfaces - Different View

	SRAFA-SFARFA																	
	S=3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
N=2	10.00	3.87	2.47	1.83	1.47	1.23	1.06	0.93	0.83	0.75	0.68	0.63	0.58	0.54	0.51	0.48	0.45	0.43
3	-9.50	21.54	42.33	-6.01	-1.97	-1.01	-0.62	-0.42	-0.31	-0.23	-0.19	-0.15	-0.12	-0.10	-0.09	-0.08	-0.07	-0.06
4	1.08	-6.67	4.91	2.19	1.32	0.95	0.75	0.62	0.53	0.46	0.41	0.37	0.34	0.31	0.29	0.27	0.25	0.24
5	-1.00	1.45	-6.62	36.16	-8.22	-1.52	-0.64	-0.36	-0.23	-0.16	-0.12	-0.09	-0.08	-0.06	-0.05	-0.04	-0.04	-0.03
6	-1.00	-1.08	1.64	-7.60	-108.82	1.03	0.76	0.57	0.46	0.38	0.33	0.29	0.26	0.23	0.21	0.20	0.18	0.17
7	-1.00	-1.08	-1.16	1.74	-5.66	47.90	-4.85	-0.96	-0.40	-0.22	-0.14	-0.10	-0.07	-0.06	-0.05	-0.04	-0.03	-0.03
8	-1.00	-1.08	-1.16	-1.23	1.81	-7.14	98.24	0.16	0.43	0.36	0.30	0.26	0.23	0.20	0.18	0.16	0.15	0.14
9	-1.00	-1.08	-1.16	-1.23	-1.29	1.85	-5.20	57.25	-4.07	-0.77	-0.31	-0.17	-0.10	-0.07	-0.05	-0.04	-0.03	-0.03
10	-1.00	-1.08	-1.16	-1.23	-1.29	-1.35	1.88	-6.89	78.88	-0.51	0.21	0.23	0.21	0.18	0.16	0.15	0.13	0.12
11	-1.00	-1.08	-1.16	-1.23	-1.29	-1.35	-1.40	1.90	-4.94	65.57	-3.82	-0.70	-0.27	-0.14	-0.09	-0.06	-0.04	-0.03
12	-1.00	-1.08	-1.16	-1.23	-1.29	-1.35	-1.40	-1.44	1.92	-4.73	79.33	-1.06	0.05	0.14	0.15	0.13	0.12	0.11
13	-1.00	-1.08	-1.16	-1.23	-1.29	-1.35	-1.40	-1.44	-1.47	1.93	-4.77	73.51	-3.76	-0.66	-0.25	-0.13	-0.08	-0.05
14	-1.00	-1.08	-1.16	-1.23	-1.29	-1.35	-1.40	-1.44	-1.47	-1.51	1.94	-4.62	83.78	-1.51	-0.07	0.08	0.10	0.10
15	-1.00	-1.08	-1.16	-1.23	-1.29	-1.35	-1.40	-1.44	-1.47	-1.51	-1.53	1.94	-4.65	81.31	-3.79	-0.65	-0.24	-0.12
16	-1.00	-1.08	-1.16	-1.23	-1.29	-1.35	-1.40	-1.44	-1.47	-1.51	-1.53	-1.56	1.95	-6.53	89.71	-1.89	-0.16	0.03
17	-1.00	-1.08	-1.16	-1.23	-1.29	-1.35	-1.40	-1.44	-1.47	-1.51	-1.53	-1.56	-1.58	1.95	-4.56	89.07	-3.85	-0.65
18	-1.00	-1.08	-1.16	-1.23	-1.29	-1.35	-1.40	-1.44	-1.47	-1.51	-1.53	-1.56	-1.58	-1.60	1.96	-4.47	96.34	-2.21
19	-1.00	-1.08	-1.16	-1.23	-1.29	-1.35	-1.40	-1.44	-1.47	-1.51	-1.53	-1.56	-1.58	-1.60	-1.62	1.96	-4.50	96.84
20	-1.00	-1.08	-1.16	-1.23	-1.29	-1.35	-1.40	-1.44	-1.47	-1.51	-1.53	-1.56	-1.58	-1.60	-1.62	-1.64	1.97	-4.42

Figure 27: Table of Difference between Supplier RA/FA and FA/RFA Indifference Surfaces

	SRAFA-SRARFA																			
	S=3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20		
N=2	4.00	0.57	0.19	0.09	0.05	0.03	0.02	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00		
3	-92.00	11.50	2.74	1.50	1.04	0.80	0.66	0.56	0.48	0.43	0.39	0.35	0.32	0.30	0.28	0.26	0.24	0.23		
4	0.00	7.73	7.33	0.58	0.18	0.08	0.04	0.03	0.02	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00		
5	0.00	0.00	8.37	14.52	1.92	0.94	0.63	0.48	0.39	0.33	0.29	0.26	0.23	0.21	0.19	0.18	0.17	0.16		
6	0.00	0.00	0.00	4.18	11.31	0.60	0.17	0.07	0.04	0.02	0.02	0.01	0.01	0.01	0.00	0.00	0.00	0.00		
7	0.00	0.00	0.00	0.00	4.22	18.00	1.47	0.65	0.42	0.32	0.26	0.22	0.19	0.17	0.15	0.14	0.13	0.12		
8	0.00	0.00	0.00	0.00	0.00	3.05	15.74	0.61	0.17	0.07	0.04	0.02	0.02	0.01	0.01	0.01	0.00	0.00		
9	0.00	0.00	0.00	0.00	0.00	0.00	3.01	21.97	1.22	0.50	0.31	0.23	0.18	0.16	0.14	0.12	0.11	0.10		
10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.48	20.49	0.62	0.17	0.07	0.04	0.02	0.01	0.01	0.01	0.01		
11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.43	26.35	1.06	0.41	0.24	0.18	0.14	0.12	0.10	0.09		
12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.14	25.50	0.63	0.17	0.07	0.04	0.02	0.01	0.01		
13	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.08	31.06	0.96	0.35	0.20	0.14	0.11	0.09		
14	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.90	30.71	0.63	0.17	0.07	0.04	0.02		
15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.85	36.02	0.89	0.30	0.17	0.12		
16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.72	36.09	0.63	0.17	0.07		
17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.68	41.19	0.83	0.27		
18	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.59	41.61	0.62		
19	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.55	46.54		
20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.48		

							SRARFA-SFARFA															
	S=3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20				
N=2	6.00	3.29	2.28	1.75	1.42	1.20	1.04	0.92	0.82	0.74	0.68	0.62	0.58	0.54	0.51	0.47	0.45	0.42				
3	82.50	10.04	39.60	-7.52	-3.01	-1.81	-1.28	-0.98	-0.79	-0.66	-0.57	-0.50	-0.44	-0.40	-0.36	-0.33	-0.31	-0.29				
4	1.08	-16.40	-2.42	1.61	1.15	0.88	0.71	0.60	0.51	0.45	0.40	0.37	0.33	0.31	0.29	0.27	0.25	0.23				
5	-1.00	1.45	-15.00	21.64	-10.14	-2.45	-1.27	-0.83	-0.62	-0.49	-0.41	-0.35	-0.31	-0.27	-0.25	-0.22	-0.21	-0.19				
6	-1.00	-1.08	1.64	-11.78	-120.14	0.43	0.59	0.50	0.42	0.36	0.31	0.28	0.25	0.23	0.21	0.19	0.18	0.17				
7	-1.00	-1.08	-1.16	1.74	-9.88	29.90	-6.33	-1.61	-0.82	-0.54	-0.40	-0.32	-0.26	-0.23	-0.20	-0.18	-0.16	-0.15				
8	-1.00	-1.08	-1.16	-1.23	1.81	-10.19	82.50	-0.45	0.26	0.29	0.27	0.24	0.21	0.19	0.17	0.16	0.15	0.14				
9	-1.00	-1.08	-1.16	-1.23	-1.29	1.85	-8.21	35.28	-5.28	-1.27	-0.62	-0.40	-0.29	-0.23	-0.19	-0.16	-0.14	-0.13				
10	-1.00	-1.08	-1.16	-1.23	-1.29	-1.35	1.88	-9.37	58.39	-1.14	0.04	0.16	0.17	0.16	0.15	0.14	0.13	0.12				
11	-1.00	-1.08	-1.16	-1.23	-1.29	-1.35	-1.40	1.90	-7.37	39.22	-4.88	-1.10	-0.52	-0.32	-0.23	-0.18	-0.15	-0.13				
12	-1.00	-1.08	-1.16	-1.23	-1.29	-1.35	-1.40	-1.44	1.92	-6.86	53.83	-1.68	-0.12	0.07	0.11	0.11	0.11	0.10				
13	-1.00	-1.08	-1.16	-1.23	-1.29	-1.35	-1.40	-1.44	-1.47	1.93	-6.85	42.45	-4.72	-1.01	-0.45	-0.27	-0.19	-0.15				
14	-1.00	-1.08	-1.16	-1.23	-1.29	-1.35	-1.40	-1.44	-1.47	-1.51	1.94	-6.51	53.07	-2.14	-0.24	0.01	0.06	0.08				
15	-1.00	-1.08	-1.16	-1.23	-1.29	-1.35	-1.40	-1.44	-1.47	-1.51	-1.53	1.94	-6.50	45.29	-4.67	-0.96	-0.41	-0.24				
16	-1.00	-1.08	-1.16	-1.23	-1.29	-1.35	-1.40	-1.44	-1.47	-1.51	-1.53	-1.56	1.95	-8.26	53.62	-2.51	-0.33	-0.04				
17	-1.00	-1.08	-1.16	-1.23	-1.29	-1.35	-1.40	-1.44	-1.47	-1.51	-1.53	-1.56	-1.58	1.95	-6.24	47.88	-4.68	-0.92				
18	-1.00	-1.08	-1.16	-1.23	-1.29	-1.35	-1.40	-1.44	-1.47	-1.51	-1.53	-1.56	-1.58	-1.60	1.96	-6.06	54.74	-2.84				
19	-1.00	-1.08	-1.16	-1.23	-1.29	-1.35	-1.40	-1.44	-1.47	-1.51	-1.53	-1.56	-1.58	-1.60	-1.62	1.96	-6.04	50.30				
20	-1.00	-1.08	-1.16	-1.23	-1.29	-1.35	-1.40	-1.44	-1.47	-1.51	-1.53	-1.56	-1.58	-1.60	-1.62	-1.64	1.97	-5.90				

Figure 28: Tables of Difference between Supplier Indifference Surfaces

		BRAFA-BFARFA																		
	S=3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20		
N=2	-13.00	19.00	6.50	4.25	3.31	2.79	2.46	2.24	2.07	1.94	1.84	1.76	1.70	1.64	1.59	1.55	1.52	1.49		
3	-5.50	-60.00	2.25	-0.76	-1.25	-1.36	-1.38	-1.37	-1.35	-1.33	-1.31	-1.29	-1.27	-1.25	-1.24	-1.23	-1.22	-1.20		
4	-0.51	-3.06	52.00	7.32	-14.53	-7.60	-5.49	-4.51	-3.95	-3.58	-3.33	-3.15	-3.01	-2.90	-2.81	-2.74	-2.68	-2.62		
5	6.00	35.08	-27.50	7.95	0.84	-0.12	-0.41	-0.52	-0.57	-0.59	-0.60	-0.60	-0.60	-0.60	-0.60	-0.59	-0.59	-0.59		
6	0.00	1.00	-1.36	-26.31	55.63	-12.05	-4.96	-3.27	-2.55	-2.17	-1.93	-1.77	-1.65	-1.57	-1.50	-1.45	-1.41	-1.37		
7	0.00	0.00	11.50	-27.86	100.49	2.40	0.46	-0.02	-0.20	-0.28	-0.33	-0.35	-0.36	-0.37	-0.38	-0.38	-0.38	-0.38		
8	0.00	0.00	0.00	1.00	-3.02	35.39	-20.09	-4.68	-2.67	-1.96	-1.60	-1.39	-1.25	-1.16	-1.09	-1.04	-1.00	-0.96		
9	0.00	0.00	0.00	0.00	55.00	-18.35	7.74	1.22	0.31	0.01	-0.12	-0.18	-0.22	-0.24	-0.25	-0.26	-0.27	-0.27		
10	0.00	0.00	0.00	0.00	0.00	1.00	-7.96	70.41	-6.18	-2.64	-1.74	-1.35	-1.14	-1.01	-0.92	-0.85	-0.81	-0.77		
11	0.00	0.00	0.00	0.00	0.00	0.00	-24.38	-27.78	3.27	0.76	0.22	0.02	-0.08	-0.13	-0.16	-0.18	-0.19	-0.20		
12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	-52.55	-20.37	-3.26	-1.75	-1.25	-1.01	-0.87	-0.78	-0.72	-0.67		
13	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-10.82	87.07	1.89	0.54	0.17	0.02	-0.05	-0.10	-0.12	-0.14		
14	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	37.43	-6.80	-2.10	-1.28	-0.96	-0.80	-0.70	-0.63		
15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-7.27	12.22	1.27	0.40	0.14	0.02	-0.04	-0.08		
16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	32.42	-3.69	-1.50	-0.99	-0.77	-0.65		
17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-5.63	5.87	0.93	0.32	0.11	0.02		
18	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	215.37	-2.41	-1.15	-0.80		
19	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-4.69	3.65	0.72	0.26	
20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	-22.39	-1.74		

	BRAFA-BRARFA																		
	S=3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
N=2	-6.00	19.00	-2.50	-0.75	-0.36	-0.21	-0.14	-0.10	-0.07	-0.06	-0.05	-0.04	-0.03	-0.03	-0.02	-0.02	-0.02	-0.01	
3	-1.50	-36.00	10.25	-8.76	-4.25	-2.96	-2.38	-2.05	-1.85	-1.71	-1.61	-1.53	-1.47	-1.42	-1.38	-1.35	-1.32	-1.30	
4	-0.65	-3.06	70.00	-3.68	-0.89	-0.40	-0.23	-0.15	-0.10	-0.08	-0.06	-0.05	-0.04	-0.03	-0.03	-0.02	-0.02	-0.02	
5	-0.50	-0.92	-7.50	53.41	-8.16	-3.00	-1.86	-1.40	-1.17	-1.02	-0.93	-0.86	-0.81	-0.77	-0.74	-0.72	-0.70	-0.68	
6	0.00	-0.80	-1.24	-23.31	-9.69	-1.25	-0.48	-0.26	-0.16	-0.11	-0.08	-0.06	-0.05	-0.04	-0.03	-0.03	-0.02	-0.02	
7	0.00	0.00	-1.23	-1.92	119.39	-15.40	-3.05	-1.61	-1.12	-0.89	-0.76	-0.67	-0.61	-0.57	-0.54	-0.51	-0.50	-0.48	
8	0.00	0.00	0.00	-1.55	-2.61	71.39	-2.40	-0.66	-0.31	-0.18	-0.12	-0.09	-0.06	-0.05	-0.04	-0.03	-0.03	-0.02	
9	0.00	0.00	0.00	0.00	-2.33	-4.44	68.02	-4.42	-1.69	-1.04	-0.77	-0.63	-0.55	-0.49	-0.45	-0.43	-0.40	-0.39	
10	0.00	0.00	0.00	0.00	0.00	-2.70	-7.00	-12.66	-1.15	-0.43	-0.23	-0.14	-0.10	-0.07	-0.05	-0.04	-0.03	-0.03	
11	0.00	0.00	0.00	0.00	0.00	0.00	-4.12	-15.26	-16.88	-2.26	-1.11	-0.75	-0.58	-0.49	-0.43	-0.39	-0.36	-0.34	
12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-4.58	-50.20	-3.59	-0.70	-0.30	-0.17	-0.11	-0.08	-0.06	-0.05	-0.04	
13	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-7.59	101.63	-5.31	-1.42	-0.80	-0.57	-0.46	-0.39	-0.35	-0.32	
14	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-8.25	45.24	-1.79	-0.48	-0.23	-0.14	-0.09	-0.07	-0.05	
15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-17.28	35.65	-2.79	-1.01	-0.62	-0.46	-0.38	-0.32	
16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-18.50	196.40	-1.11	-0.36	-0.18	-0.11	-0.08	
17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-193.95	205.44	-1.79	-0.76	-0.50	-0.38	
18	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-205.42	-17.61	-0.78	-0.28	-0.15	
19	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	29.04	-20.14	-1.27	-0.61	
20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	30.52	-6.50	-0.58	

Figure 29: Table of Difference between Buyer Indifference Surfaces

		BRARFA-BFARFA																					
	S=3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20					
N=2	-7.00	0.00	9.00	5.00	3.67	3.00	2.60	2.33	2.14	2.00	1.89	1.80	1.73	1.67	1.62	1.57	1.53	1.50					
3	6.00	-10.00	-14.00	6.00	2.00	1.00	0.60	0.40	0.29	0.21	0.17	0.13	0.11	0.09	0.08	0.07	0.06	0.05					
4	2.80	8.00	-31.00	-8.00	-5.64	-4.53	-3.94	-3.56	-3.31	-3.13	-2.99	-2.88	-2.79	-2.72	-2.66	-2.61	-2.57	-2.53					
5	3.00	3.50	32.50	-52.50	7.05	1.95	0.92	0.54	0.35	0.25	0.18	0.14	0.11	0.09	0.08	0.06	0.06	0.05					
6	0.00	3.60	4.09	36.00	2.32	-2.35	-2.13	-1.91	-1.75	-1.64	-1.55	-1.49	-1.44	-1.39	-1.36	-1.33	-1.30	-1.28					
7	0.00	0.00	4.47	8.27	-34.21	15.31	2.49	1.03	0.56	0.36	0.25	0.18	0.14	0.11	0.09	0.07	0.06	0.05					
8	0.00	0.00	0.00	5.11	9.04	-75.10	-0.20	-1.17	-1.19	-1.13	-1.07	-1.02	-0.98	-0.95	-0.92	-0.90	-0.88	-0.87					
9	0.00	0.00	0.00	0.00	6.65	50.67	-64.58	4.30	1.34	0.66	0.39	0.26	0.19	0.14	0.11	0.09	0.07	0.06					
10	0.00	0.00	0.00	0.00	0.00	7.39	55.35	11.32	-0.25	-0.73	-0.78	-0.77	-0.75	-0.73	-0.71	-0.69	-0.67	-0.66					
11	0.00	0.00	0.00	0.00	0.00	0.00	10.24	-30.50	17.99	2.16	0.86	0.47	0.30	0.21	0.15	0.12	0.09	0.08					
12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	11.17	-35.62	2.84	-0.21	-0.51	-0.57	-0.57	-0.55	-0.54	-0.53	-0.52					
13	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	17.19	-20.41	5.85	1.35	0.61	0.36	0.24	0.17	0.13	0.10					
14	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	18.51	-29.79	1.29	-0.17	-0.38	-0.44	-0.45	-0.45	-0.44					
15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	36.56	-26.55	3.12	0.94	0.47	0.29	0.19	0.14					
16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	39.00	-192.30	0.75	-0.14	-0.30	-0.35	-0.37					
17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	389.90	-201.58	2.01	0.71	0.37	0.24					
18	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	412.84	19.55	0.49	-0.12	-0.25					
19	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-56.09	22.36	1.43	0.56					
20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-59.04	7.65	0.35					

Figure 30: Table of Difference between Buyer RA/RFA AND FA/RFA Indifference Surfaces

		SPARFA-BRARFA																		
	S=3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20		
N=2	18.00	7.00	-3.33	0.00	0.93	1.33	1.54	1.67	1.75	1.80	1.84	1.87	1.89	1.90	1.92	1.93	1.94	1.94		
3	87.00	17.50	5.26	-9.50	-3.89	-2.12	-1.27	-0.78	-0.46	-0.24	-0.07	0.05	0.15	0.23	0.30	0.36	0.40	0.45		
4	3.20	-10.40	46.67	-3.25	0.57	1.33	1.61	1.74	1.82	1.86	1.89	1.91	1.93	1.94	1.95	1.96	1.96	1.97		
5	0.50	3.50	-9.37	50.98	-7.32	-1.69	-0.38	0.18	0.49	0.68	0.81	0.90	0.97	1.03	1.07	1.11	1.14	1.17		
6	1.00	0.20	3.91	-0.78	0.36	2.02	2.04	2.03	2.02	2.01	2.01	2.01	2.01	2.00	2.00	2.00	2.00	2.00		
7	1.00	1.00	-0.23	4.23	0.95	-5.71	0.13	0.77	1.01	1.14	1.23	1.28	1.33	1.36	1.39	1.41	1.43	1.45		
8	1.00	1.00	1.00	-0.55	4.96	39.81	14.86	3.96	2.82	2.46	2.30	2.21	2.15	2.12	2.09	2.08	2.06	2.05		
9	1.00	1.00	1.00	1.00	-1.33	5.33	45.99	11.45	2.79	1.99	1.77	1.69	1.65	1.64	1.63	1.63	1.63	1.63		
10	1.00	1.00	1.00	1.00	1.00	-1.70	6.65	-30.45	20.78	5.02	3.32	2.76	2.50	2.35	2.26	2.21	2.17	2.14		
11	1.00	1.00	1.00	1.00	1.00	1.00	-3.12	7.12	-32.61	18.25	4.20	2.72	2.25	2.04	1.93	1.87	1.83	1.81		
12	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-3.58	9.93	-17.77	24.99	5.70	3.67	2.98	2.65	2.47	2.35	2.28		
13	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-6.59	10.59	-18.56	22.98	5.09	3.21	2.58	2.29	2.13	2.04		
14	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-7.25	19.06	-14.23	28.48	6.18	3.92	3.15	2.77	2.56			
15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-16.28	20.28	-14.69	26.83	5.71	3.56	2.83	2.48			
16	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-17.50	185.48	-12.50	31.56	6.55	4.12	3.28			
17	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-192.95	196.95	-12.81	30.18	6.16	3.82		
18	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-204.42	-24.57	-11.45	34.36	6.83		
19	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	30.04	-26.04	-11.68	33.20		
20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	31.52	-11.63	-10.73		

	SFARFA-BFARFA																		
	S=3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
N=2	5.00	3.71	3.39	3.25	3.18	3.13	3.10	3.08	3.07	3.06	3.05	3.04	3.04	3.03	3.03	3.03	3.02	3.02	
3	10.50	-2.54	-48.33	4.01	1.12	0.69	0.60	0.60	0.62	0.64	0.66	0.69	0.71	0.73	0.74	0.76	0.77	0.78	
4	4.92	14.00	18.09	-12.86	-6.21	-4.08	-3.04	-2.42	-2.01	-1.72	-1.50	-1.33	-1.20	-1.09	-1.00	-0.92	-0.85	-0.80	
5	4.50	5.55	38.12	-23.16	9.87	2.72	1.81	1.55	1.46	1.42	1.40	1.39	1.39	1.39	1.39	1.40	1.40	1.40	
6	2.00	4.88	6.36	47.00	122.82	-0.76	-0.69	-0.39	-0.15	0.02	0.14	0.24	0.32	0.38	0.44	0.48	0.52	0.55	
7	2.00	2.08	5.39	10.76	-23.38	-20.30	8.94	3.41	2.40	2.04	1.87	1.78	1.73	1.70	1.68	1.67	1.66	1.65	
8	2.00	2.08	2.16	5.78	12.19	-25.10	-67.84	3.24	1.37	1.04	0.96	0.95	0.96	0.98	1.00	1.02	1.03	1.05	
9	2.00	2.08	2.16	2.23	6.62	54.15	-10.38	-19.53	9.41	3.92	2.78	2.35	2.13	2.01	1.93	1.88	1.85	1.82	
10	2.00	2.08	2.16	2.23	2.29	7.05	60.12	-11.75	-37.86	5.42	2.50	1.83	1.58	1.47	1.41	1.38	1.37	1.36	
11	2.00	2.08	2.16	2.23	2.29	2.35	8.51	-25.28	-7.26	-18.81	9.94	4.29	3.06	2.57	2.31	2.17	2.07	2.01	
12	2.00	2.08	2.16	2.23	2.29	2.35	2.40	9.02	-27.61	-6.07	-29.05	6.87	3.22	2.33	1.98	1.80	1.70	1.64	
13	2.00	2.08	2.16	2.23	2.29	2.35	2.40	2.44	12.07	-11.74	-5.86	-18.12	10.43	4.57	3.27	2.73	2.45	2.28	
14	2.00	2.08	2.16	2.23	2.29	2.35	2.40	2.44	12.76	-12.66	-6.42	-24.76	7.93	3.72	2.69	2.26	2.04		
15	2.00	2.08	2.16	2.23	2.29	2.35	2.40	2.44	2.47	2.51	21.81	-8.21	-5.07	-17.52	10.85	4.80	3.43	2.86	
16	2.00	2.08	2.16	2.23	2.29	2.35	2.40	2.44	2.47	2.51	2.53	23.06	-8.77	-3.49	-22.20	8.76	4.10	2.96	
17	2.00	2.08	2.16	2.23	2.29	2.35	2.40	2.44	2.47	2.51	2.53	2.56	198.53	-6.58	-4.56	-16.99	11.22	4.98	
18	2.00	2.08	2.16	2.23	2.29	2.35	2.40	2.44	2.47	2.51	2.53	2.56	2.58	210.02	-6.98	-4.89	-20.49	9.42	
19	2.00	2.08	2.16	2.23	2.29	2.35	2.40	2.44	2.47	2.51	2.53	2.56	2.58	2.60	-24.42	-5.65	-4.20	-16.54	
20	2.00	2.08	2.16	2.23	2.29	2.35	2.40	2.44	2.47	2.51	2.53	2.56	2.58	2.60	2.62	-25.88	-5.95	-4.48	

Figure 31: Tables of Difference between Supplier and Buyer RA/RFA and FA/RFA Indifference Surfaces

										SRAFA-BRAFA																		
	S=3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20										
N=2	28.00	-11.43	-0.64	0.84	1.34	1.57	1.70	1.78	1.83	1.86	1.89	1.91	1.92	1.93	1.94	1.95	1.96	1.96										
3	-3.50	65.00	-2.25	0.76	1.40	1.64	1.76	1.83	1.87	1.90	1.92	1.93	1.94	1.95	1.96	1.96	1.97	1.97										
4	3.85	0.39	-16.00	1.02	1.64	1.81	1.88	1.91	1.94	1.95	1.96	1.97	1.97	1.98	1.98	1.98	1.98	1.98										
5	1.00	4.42	6.50	12.09	2.76	2.24	2.11	2.06	2.04	2.03	2.02	2.01	2.01	2.01	2.01	2.01	2.00	2.00										
6	1.00	1.00	5.16	26.71	21.37	3.87	2.69	2.36	2.22	2.15	2.10	2.08	2.06	2.05	2.04	2.03	2.03	2.02										
7	1.00	1.00	1.00	6.15	-114.22	27.69	4.65	3.03	2.55	2.35	2.24	2.17	2.13	2.10	2.08	2.07	2.06	2.05										
8	1.00	1.00	1.00	1.00	7.57	-28.53	33.00	5.24	3.30	2.72	2.46	2.32	2.23	2.18	2.14	2.12	2.10	2.08										
9	1.00	1.00	1.00	1.00	1.00	9.77	-19.02	37.84	5.69	3.53	2.85	2.55	2.39	2.29	2.22	2.18	2.14	2.12										
10	1.00	1.00	1.00	1.00	1.00	1.00	13.66	-15.30	42.42	6.06	3.71	2.97	2.63	2.45	2.33	2.26	2.21	2.17										
11	1.00	1.00	1.00	1.00	1.00	1.00	1.00	22.38	-13.30	46.86	6.37	3.87	3.07	2.70	2.50	2.38	2.29	2.24										
12	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	60.13	-12.05	51.19	6.63	4.01	3.16	2.77	2.55	2.42	2.33										
13	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-91.03	-11.18	55.46	6.85	4.12	3.24	2.83	2.60	2.45										
14	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-26.18	-10.54	59.68	7.04	4.23	3.31	2.88	2.64											
15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-15.37	-10.05	63.86	7.21	4.32	3.37	2.92										
16	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-10.92	-9.66	68.01	7.36	4.40	3.43										
17	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-8.49	-9.34	72.14	7.49	4.47										
18	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-6.96	-9.08	76.25	7.61											
19	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-5.90	-8.86	80.35										
20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-5.13	-8.67										

										SFAFA-BRAFA																		
	S=3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20										
N=2	12.00	3.71	-5.61	-1.75	-0.49	0.13	0.50	0.75	0.93	1.06	1.16	1.24	1.31	1.37	1.41	1.45	1.49	1.52										
3	4.50	7.46	-34.33	-1.99	-0.88	-0.31	0.00	0.20	0.33	0.43	0.50	0.55	0.60	0.63	0.67	0.69	0.71	0.73										
4	2.12	6.00	49.09	-4.86	-0.58	0.45	0.90	1.15	1.30	1.41	1.49	1.55	1.60	1.63	1.67	1.69	1.71	1.73										
5	1.50	2.05	5.62	29.34	2.82	0.76	0.89	1.02	1.11	1.17	1.21	1.25	1.28	1.30	1.32	1.33	1.35	1.36										
6	2.00	1.28	2.27	11.00	120.50	1.59	1.44	1.53	1.60	1.65	1.70	1.73	1.75	1.78	1.79	1.81	1.82	1.83										
7	2.00	2.08	0.92	2.49	10.83	-35.61	6.45	2.38	1.83	1.68	1.62	1.60	1.59	1.59	1.59	1.59	1.59	1.60										
8	2.00	2.08	2.16	0.68	3.15	50.00	-67.64	4.41	2.56	2.17	2.03	1.97	1.94	1.93	1.92	1.92	1.92	1.92										
9	2.00	2.08	2.16	2.23	-0.03	3.48	54.20	-23.83	8.07	3.26	2.39	2.08	1.94	1.86	1.82	1.79	1.77	1.76										
10	2.00	2.08	2.16	2.23	2.29	-0.35	4.77	-23.08	-37.61	6.15	3.28	2.60	2.33	2.19	2.12	2.07	2.04	2.02										
11	2.00	2.08	2.16	2.23	2.29	2.35	-1.72	5.22	-25.25	-20.97	9.08	3.82	2.76	2.36	2.16	2.05	1.98	1.93										
12	2.00	2.08	2.16	2.23	2.29	2.35	2.40	-2.15	8.02	-8.91	-28.84	7.38	3.78	2.91	2.54	2.36	2.25	2.18										
13	2.00	2.08	2.16	2.23	2.29	2.35	2.40	2.44	-5.12	8.67	-11.72	-19.47	9.81	4.22	3.03	2.56	2.33	2.19										
14	2.00	2.08	2.16	2.23	2.29	2.35	2.40	2.44	2.47	-5.75	17.13	-7.72	-24.59	8.32	4.16	3.14	2.71	2.48										
15	2.00	2.08	2.16	2.23	2.29	2.35	2.40	2.44	2.47	2.51	-14.75	18.34	-8.19	-18.46	10.38	4.51	3.24	2.72										
16	2.00	2.08	2.16	2.23	2.29	2.35	2.40	2.44	2.47	2.51	2.53	-15.94	183.53	-4.24	-22.06	9.06	4.45	3.32										
17	2.00	2.08	2.16	2.23	2.29	2.35	2.40	2.44	2.47	2.51	2.53	2.56	-191.37	195.00	-6.57	-17.70	10.84	4.74										
18	2.00	2.08	2.16	2.23	2.29	2.35	2.40	2.44	2.47	2.51	2.53	2.56	2.58	-202.82	-26.53	-5.39	-20.37	9.67										
19	2.00	2.08	2.16	2.23	2.29	2.35	2.40	2.44	2.47	2.51	2.53	2.56	2.58	2.60	31.67	-28.01	-5.63	-17.09										
20	2.00	2.08	2.16	2.23	2.29	2.35	2.40	2.44	2.47	2.51	2.53	2.56	2.58	2.60	2.62	33.16	-13.60	-4.83										

Figure 32: Tables of Difference between Supplier and Buyer Indifference Surfaces

	SFARFA-BRAFA																			
	S=3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20		
N=2	18.00	-15.29	-3.11	-1.00	-0.13	0.34	0.64	0.85	1.00	1.11	1.12	1.13	1.20	1.28	1.34	1.39	1.43	1.47	1.51	1.54
3	6.00	43.46	-44.58	6.78	3.37	2.65	2.38	2.25	2.18	2.13	2.10	2.08	2.07	2.06	2.05	2.04	2.04	2.03		
4	2.77	9.06	-20.91	-1.18	0.31	0.85	1.13	1.29	1.41	1.49	1.55	1.60	1.63	1.67	1.69	1.71	1.73	1.75		
5	2.00	2.97	13.12	-24.07	10.98	3.76	2.76	2.42	2.27	2.19	2.14	2.11	2.09	2.07	2.06	2.05	2.04	2.04		
6	2.00	2.08	3.51	34.31	130.19	2.84	1.93	1.78	1.76	1.76	1.78	1.79	1.80	1.81	1.83	1.84	1.85	1.85		
7	2.00	2.08	2.16	4.41	-108.56	-20.21	9.50	3.99	2.95	2.57	2.38	2.27	2.21	2.16	2.13	2.11	2.09	2.08		
8	2.00	2.08	2.16	2.23	5.76	-21.39	-65.24	5.07	2.87	2.35	2.15	2.06	2.01	1.98	1.96	1.95	1.95	1.94		
9	2.00	2.08	2.16	2.23	2.29	7.93	-13.81	-19.42	9.76	4.30	3.16	2.72	2.49	2.36	2.27	2.22	2.18	2.15		
10	2.00	2.08	2.16	2.23	2.29	2.35	11.78	-10.41	-36.46	6.58	3.50	2.74	2.42	2.26	2.17	2.11	2.07	2.05		
11	2.00	2.08	2.16	2.23	2.29	2.35	2.40	20.48	-8.36	-18.71	10.19	4.57	3.34	2.85	2.59	2.44	2.34	2.27		
12	2.00	2.08	2.16	2.23	2.29	2.35	2.40	2.44	58.22	-5.32	-28.14	7.69	3.96	3.02	2.62	2.42	2.29	2.21		
13	2.00	2.08	2.16	2.23	2.29	2.35	2.40	2.44	2.47	-92.96	-6.41	-18.04	10.61	4.79	3.49	2.96	2.67	2.50		
14	2.00	2.08	2.16	2.23	2.29	2.35	2.40	2.44	2.47	2.51	-28.11	-5.92	-24.10	8.55	4.30	3.23	2.78	2.54		
15	2.00	2.08	2.16	2.23	2.29	2.35	2.40	2.44	2.47	2.51	2.53	-17.31	-5.40	-17.45	11.00	4.97	3.62	3.05		
16	2.00	2.08	2.16	2.23	2.29	2.35	2.40	2.44	2.47	2.51	2.53	2.56	-12.87	-3.13	-21.70	9.24	4.56	3.40		
17	2.00	2.08	2.16	2.23	2.29	2.35	2.40	2.44	2.47	2.51	2.53	2.56	2.58	-10.44	-4.78	-16.93	11.34	5.12		
18	2.00	2.08	2.16	2.23	2.29	2.35	2.40	2.44	2.47	2.51	2.53	2.56	2.58	2.60	-8.92	-4.61	-20.09	9.82		
19	2.00	2.08	2.16	2.23	2.29	2.35	2.40	2.44	2.47	2.51	2.53	2.56	2.58	2.60	2.62	-7.86	-4.36	-16.49		
20	2.00	2.08	2.16	2.23	2.29	2.35	2.40	2.44	2.47	2.51	2.53	2.56	2.58	2.60	2.62	2.64	-7.10	-4.25		

	SPARFA-BFARFA																		
	S=3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
N=2	11.00	7.00	5.67	5.00	4.60	4.33	4.14	4.00	3.89	3.80	3.73	3.67	3.62	3.57	3.53	3.50	3.47	3.44	
3	93.00	7.50	-8.74	-3.50	-1.89	-1.12	-0.67	-0.38	-0.18	-0.02	0.09	0.19	0.26	0.33	0.38	0.42	0.46	0.50	
4	6.00	-2.40	15.67	-11.25	-5.06	-3.20	-2.33	-1.82	-1.49	-1.26	-1.10	-0.97	-0.86	-0.78	-0.71	-0.65	-0.61	-0.56	
5	3.50	7.00	23.13	-1.52	-0.27	0.26	0.54	0.72	0.84	0.92	0.99	1.04	1.08	1.12	1.15	1.17	1.20	1.21	
6	1.00	3.80	8.00	35.22	2.69	-0.33	-0.10	0.11	0.27	0.38	0.46	0.52	0.57	0.61	0.65	0.67	0.70	0.72	
7	1.00	1.00	4.23	12.50	-33.26	9.60	2.61	1.79	1.57	1.50	1.47	1.47	1.47	1.47	1.48	1.49	1.49	1.50	
8	1.00	1.00	1.00	4.55	14.00	-35.29	14.66	2.78	1.63	1.33	1.23	1.19	1.17	1.17	1.17	1.17	1.18	1.19	
9	1.00	1.00	1.00	1.00	5.33	56.00	-18.59	15.75	4.12	2.64	2.16	1.95	1.84	1.78	1.74	1.72	1.70	1.69	
10	1.00	1.00	1.00	1.00	1.00	5.70	62.00	-19.13	20.53	4.29	2.54	1.99	1.75	1.63	1.56	1.52	1.49	1.48	
11	1.00	1.00	1.00	1.00	1.00	1.00	7.12	-23.38	-14.62	20.41	5.06	3.19	2.55	2.25	2.09	1.99	1.92	1.88	
12	1.00	1.00	1.00	1.00	1.00	1.00	1.00	7.58	-25.69	-14.93	24.79	5.19	3.10	2.40	2.09	1.91	1.81	1.75	
13	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	10.59	-9.82	-12.71	24.33	5.70	3.56	2.82	2.46	2.26	2.14	
14	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	11.25	-10.73	-12.94	28.31	5.80	3.48	2.70	2.32	2.12	
15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	20.28	-6.27	-11.57	27.77	6.17	3.84	3.02	2.62	
16	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	21.50	-6.82	-11.75	31.42	6.24	3.77	2.91	
17	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	196.95	-4.63	-10.80	30.89	6.54	4.06	
18	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	208.42	-5.02	-10.95	34.25	6.58	
19	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-26.04	-3.69	-10.25	33.78	
20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-27.52	-3.98	-10.38	

Figure 33: Tables of Difference between Supplier and Buyer Indifference Surfaces

	SRAFA-BRARFA																			
	S=3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20		
N=2	22.00	7.57	-3.14	0.09	0.98	1.36	1.56	1.68	1.76	1.81	1.84	1.87	1.89	1.91	1.92	1.93	1.94	1.95		
3	-5.00	29.00	8.00	-8.00	-2.85	-1.32	-0.62	-0.22	0.02	0.19	0.31	0.40	0.47	0.53	0.58	0.61	0.65	0.67		
4	3.20	-2.67	54.00	-2.67	0.75	1.41	1.65	1.77	1.83	1.87	1.90	1.92	1.93	1.95	1.95	1.96	1.97	1.97		
5	0.50	3.50	-1.00	65.50	-5.40	-0.75	0.25	0.66	0.87	1.01	1.09	1.15	1.20	1.24	1.27	1.29	1.31	1.32		
6	1.00	0.20	3.91	3.40	11.68	2.62	2.21	2.10	2.06	2.04	2.02	2.02	2.01	2.01	2.01	2.01	2.01	2.00		
7	1.00	1.00	-0.23	4.23	5.17	12.29	1.60	1.42	1.43	1.46	1.48	1.50	1.52	1.53	1.54	1.55	1.56	1.57		
8	1.00	1.00	1.00	-0.55	4.96	42.86	30.60	4.57	2.99	2.53	2.33	2.23	2.17	2.13	2.10	2.08	2.07	2.06		
9	1.00	1.00	1.00	1.00	-1.33	5.33	49.00	33.42	4.01	2.48	2.08	1.92	1.84	1.79	1.77	1.75	1.74	1.73		
10	1.00	1.00	1.00	1.00	1.00	-1.70	6.65	-27.97	41.28	5.64	3.49	2.83	2.54	2.38	2.28	2.22	2.17	2.14		
11	1.00	1.00	1.00	1.00	1.00	1.00	-3.12	7.12	-30.18	44.60	5.26	3.12	2.49	2.22	2.07	1.99	1.94	1.90		
12	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-3.58	9.93	-15.64	50.49	6.32	3.83	3.05	2.69	2.49	2.37	2.29		
13	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-6.59	10.59	-16.48	54.04	6.05	3.55	2.78	2.43	2.25	2.13		
14	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-7.25	19.06	-12.33	59.19	6.81	4.09	3.22	2.81	2.58		
15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-16.28	20.28	-12.84	62.85	6.59	3.86	3.00	2.60		
16	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-17.50	185.48	-10.77	67.65	7.17	4.29	3.35		
17	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-192.95	196.95	-11.13	71.38	6.99	4.09		
18	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-204.42	-24.57	-9.86	75.97	7.46	
19	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	30.04	-26.04	-10.13	79.75	
20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	31.52	-11.63	-9.25	

Figure 34: Table of Difference between Supplier RA/FA and Buyer RA/RFA Indifference Surfaces

	SRAFA-BRAFA																			
	S=3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20		
N=2	24.00	-12.00	-0.83	0.75	1.29	1.54	1.68	1.76	1.82	1.86	1.88	1.90	1.92	1.93	1.94	1.95	1.95	1.96		
3	88.50	53.50	-4.99	-0.74	0.36	0.84	1.10	1.27	1.39	1.47	1.53	1.58	1.62	1.66	1.68	1.71	1.73	1.75		
4	3.85	-7.34	-23.33	0.43	1.46	1.73	1.83	1.89	1.92	1.94	1.95	1.96	1.97	1.97	1.98	1.98	1.98	1.98		
5	1.00	4.42	-1.87	-2.43	0.84	1.31	1.49	1.59	1.65	1.70	1.73	1.76	1.78	1.80	1.81	1.83	1.84	1.85		
6	1.00	1.00	5.16	22.53	10.05	3.27	2.52	2.28	2.18	2.12	2.09	2.07	2.05	2.04	2.04	2.03	2.03	2.02		
7	1.00	1.00	1.00	6.15	-118.44	9.69	3.18	2.38	2.13	2.03	1.98	1.96	1.94	1.93	1.93	1.93	1.93	1.93		
8	1.00	1.00	1.00	1.00	7.57	-31.58	17.26	4.62	3.13	2.64	2.42	2.29	2.22	2.17	2.13	2.11	2.09	2.08		
9	1.00	1.00	1.00	1.00	1.00	9.77	-22.02	15.86	4.48	3.03	2.54	2.32	2.20	2.13	2.08	2.05	2.03	2.02		
10	1.00	1.00	1.00	1.00	1.00	1.00	13.66	-17.79	21.93	5.44	3.54	2.90	2.59	2.42	2.32	2.25	2.20	2.16		
11	1.00	1.00	1.00	1.00	1.00	1.00	1.00	22.38	-15.73	20.51	5.31	3.46	2.83	2.53	2.36	2.26	2.19	2.14		
12	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	60.13	-14.18	25.69	6.00	3.84	3.09	2.73	2.53	2.40	2.32		
13	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-91.03	-13.26	24.40	5.89	3.78	3.04	2.68	2.48	2.36		
14	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-26.18	-12.44	28.97	6.41	4.06	3.24	2.84	2.61		
15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-15.37	-11.90	27.84	6.32	4.01	3.20	2.81		
16	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-10.92	-11.38	31.92	6.73	4.24	3.36	
17	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-8.49	-11.02	30.95	6.66	4.20	
18	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-6.96	-10.67	34.65	6.98	
19	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-5.90	-10.41	33.81	
20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-5.13	-10.15	

	SRAFA-BFARFA																			
	S=3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20		
N=2	15.00	7.57	5.86	5.09	4.65	4.36	4.16	4.01	3.90	3.81	3.73	3.67	3.62	3.57	3.54	3.50	3.47	3.45		
3	1.00	19.00	-6.00	-2.00	-0.85	-0.32	-0.02	0.18	0.31	0.41	0.48	0.54	0.58	0.62	0.65	0.68	0.70	0.72		
4	6.00	5.33	23.00	-10.67	-4.89	-3.13	-2.29	-1.80	-1.48	-1.25	-1.09	-0.96	-0.86	-0.78	-0.71	-0.65	-0.60	-0.56		
5	3.50	7.00	31.50	13.00	1.65	1.20	1.17	1.19	1.22	1.25	1.28	1.30	1.31	1.33	1.34	1.35	1.36	1.37		
6	1.00	3.80	8.00	39.40	14.00	0.27	0.07	0.19	0.31	0.40	0.47	0.53	0.58	0.62	0.65	0.68	0.70	0.72		
7	1.00	1.00	4.23	12.50	-29.04	27.60	4.09	2.45	2.00	1.82	1.73	1.68	1.66	1.64	1.63	1.63	1.62	1.62		
8	1.00	1.00	1.00	4.55	14.00	-32.24	30.40	3.40	1.80	1.40	1.26	1.21	1.19	1.18	1.18	1.18	1.18	1.19		
9	1.00	1.00	1.00	1.00	5.33	56.00	-15.58	37.72	5.34	3.14	2.47	2.18	2.03	1.94	1.88	1.84	1.81	1.80		
10	1.00	1.00	1.00	1.00	1.00	5.70	62.00	-16.64	41.02	4.91	2.70	2.06	1.79	1.65	1.57	1.53	1.50	1.48		
11	1.00	1.00	1.00	1.00	1.00	1.00	7.12	-23.38	-12.20	46.76	6.12	3.59	2.79	2.42	2.23	2.11	2.03	1.98		
12	1.00	1.00	1.00	1.00	1.00	1.00	1.00	7.58	-25.69	-12.80	50.29	5.81	3.27	2.48	2.12	1.94	1.83	1.76		
13	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	10.59	-9.82	-10.63	55.38	6.66	3.91	3.02	2.60	2.37	2.23		
14	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	11.25	-10.73	-11.04	59.03	6.42	3.65	2.77	2.36	2.14		
15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	20.28	-6.27	-9.72	63.79	7.06	4.15	3.19	2.74		
16	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	21.50	-6.82	-10.03	67.51	6.87	3.93	2.98		
17	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	196.95	-4.63	-9.12	72.08	7.37	4.33		
18	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	208.42	-5.02	-9.37	75.85	7.21		
19	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-26.04	-3.69	-8.70	80.30		
20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-27.52	-3.98	-6.90		

Figure 35: Tables of Difference between Supplier and Buyer Indifference Surfaces

	FA-ExpectedProfit																		
	S=3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
N=2	0.21	0.10	0.06	0.04	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	
3	0.71	0.27	0.14	0.09	0.06	0.04	0.03	0.03	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	
4	-	0.77	0.31	0.17	0.11	0.08	0.06	0.04	0.03	0.03	0.02	0.02	0.02	0.01	0.01	0.01	0.01	0.01	
5	-1.29	-	0.81	0.34	0.19	0.13	0.09	0.07	0.05	0.04	0.03	0.03	0.02	0.02	0.02	0.02	0.01	0.01	
6	-0.79	-1.23	-	0.84	0.36	0.21	0.14	0.10	0.08	0.06	0.05	0.04	0.03	0.03	0.02	0.02	0.02	0.02	
7	-0.63	-0.73	-1.19	-	0.86	0.38	0.22	0.15	0.11	0.08	0.07	0.05	0.04	0.04	0.03	0.03	0.02	0.02	
8	-0.54	-0.57	-0.69	-1.16	-	0.88	0.39	0.23	0.16	0.12	0.09	0.07	0.06	0.05	0.04	0.04	0.03	0.03	
9	-0.49	-0.48	-0.53	-0.66	-1.14	-	0.89	0.40	0.24	0.17	0.12	0.10	0.08	0.06	0.05	0.04	0.04	0.03	
10	-0.46	-0.43	-0.44	-0.50	-0.64	-1.12	-	0.90	0.41	0.25	0.17	0.13	0.10	0.08	0.07	0.06	0.05	0.04	
11	-0.43	-0.40	-0.39	-0.41	-0.48	-0.62	-1.11	-	0.91	0.42	0.26	0.18	0.13	0.10	0.08	0.07	0.06	0.05	
12	-0.42	-0.38	-0.36	-0.36	-0.39	-0.46	-0.61	-1.10	-	0.92	0.42	0.26	0.18	0.14	0.11	0.09	0.07	0.06	
13	-0.40	-0.36	-0.34	-0.33	-0.34	-0.37	-0.44	-0.60	-1.09	-	0.92	0.43	0.27	0.19	0.14	0.11	0.09	0.08	
14	-0.39	-0.35	-0.32	-0.31	-0.31	-0.32	-0.36	-0.43	-0.59	-1.08	-	0.93	0.43	0.27	0.19	0.14	0.11	0.09	
15	-0.38	-0.33	-0.30	-0.29	-0.28	-0.29	-0.31	-0.35	-0.42	-0.58	-1.08	-	0.93	0.44	0.27	0.19	0.15	0.12	
16	-0.38	-0.33	-0.29	-0.28	-0.27	-0.27	-0.28	-0.30	-0.34	-0.42	-0.58	-1.07	-	0.94	0.44	0.28	0.20	0.15	
17	-0.37	-0.32	-0.28	-0.26	-0.25	-0.25	-0.25	-0.27	-0.29	-0.33	-0.41	-0.57	-1.07	-	0.94	0.44	0.28	0.20	
18	-0.36	-0.31	-0.28	-0.25	-0.24	-0.24	-0.24	-0.24	-0.26	-0.28	-0.33	-0.40	-0.57	-1.06	-	0.94	0.45	0.28	
19	-0.36	-0.31	-0.27	-0.25	-0.23	-0.22	-0.22	-0.22	-0.23	-0.25	-0.28	-0.32	-0.40	-0.56	-1.06	-	0.95	0.45	
20	-0.35	-0.30	-0.27	-0.24	-0.23	-0.22	-0.21	-0.21	-0.22	-0.23	-0.24	-0.27	-0.32	-0.40	-0.56	-1.06	-	0.95	

								FA-ExpectedProfit																	
	S=3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20							
N=2	0.07	0.05	0.04	0.03	0.03	0.02	0.02	0.02	0.02	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01							
3	0.10	0.08	0.06	0.05	0.04	0.04	0.03	0.03	0.03	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02							
4		0.10	0.08	0.07	0.06	0.05	0.04	0.04	0.04	0.03	0.03	0.03	0.03	0.02	0.02	0.02	0.02	0.02							
5			0.10	0.08	0.07	0.06	0.06	0.05	0.05	0.04	0.04	0.04	0.03	0.03	0.03	0.03	0.03	0.03							
6				0.10	0.09	0.08	0.07	0.06	0.05	0.05	0.05	0.04	0.04	0.04	0.04	0.04	0.03	0.03							
7					0.10	0.09	0.08	0.07	0.06	0.06	0.05	0.05	0.05	0.04	0.04	0.04	0.04	0.03							
8						0.10	0.09	0.08	0.07	0.07	0.06	0.06	0.05	0.05	0.05	0.04	0.04	0.04							
9							0.10	0.09	0.08	0.08	0.07	0.06	0.06	0.06	0.05	0.05	0.05	0.05							
10								0.10	0.09	0.08	0.08	0.07	0.07	0.06	0.06	0.06	0.05	0.05							
11									0.10	0.09	0.08	0.08	0.07	0.07	0.06	0.06	0.06	0.06							
12										0.10	0.09	0.09	0.08	0.08	0.07	0.07	0.06	0.06							
13											0.10	0.09	0.09	0.08	0.08	0.07	0.07	0.07							
14												0.10	0.09	0.09	0.08	0.08	0.07	0.07							
15													0.10	0.09	0.09	0.08	0.08	0.08							
16														0.10	0.09	0.09	0.08	0.08							
17															0.10	0.09	0.09	0.09							
18																0.10	0.09	0.09							
19																	0.10	0.10							
20																		0.10							

Figure 36: Table of Expected Profit to Supplier from RA and FA Auction

		RA-ExpectedSurplus																		
	S=3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20		
N=2	-0.08	0.05	0.13	0.19	0.23	0.26	0.29	0.31	0.33	0.34	0.35	0.36	0.37	0.38	0.39	0.39	0.40	0.40		
3	-0.33	-0.03	0.13	0.24	0.32	0.37	0.41	0.45	0.48	0.50	0.52	0.53	0.55	0.56	0.57	0.58	0.59	0.60		
4	-1.08	-0.28	0.05	0.24	0.37	0.45	0.52	0.57	0.61	0.65	0.68	0.70	0.72	0.74	0.76	0.77	0.78	0.79		
5		-1.03	-0.20	0.16	0.37	0.50	0.60	0.68	0.74	0.79	0.83	0.86	0.89	0.91	0.94	0.95	0.97	0.99		
6			-0.95	-0.09	0.28	0.50	0.65	0.76	0.85	0.91	0.97	1.01	1.05	1.08	1.11	1.13	1.15	1.17		
7				-0.84	0.03	0.42	0.65	0.81	0.93	1.02	1.09	1.15	1.20	1.24	1.27	1.31	1.33	1.36		
8					-0.72	0.17	0.57	0.81	0.98	1.10	1.20	1.27	1.34	1.39	1.43	1.47	1.51	1.53		
9						-0.58	0.32	0.73	0.98	1.15	1.28	1.38	1.46	1.53	1.58	1.63	1.67	1.71		
10							-0.43	0.48	0.90	1.15	1.33	1.47	1.57	1.65	1.72	1.78	1.83	1.87		
11								-0.27	0.65	1.07	1.33	1.52	1.65	1.76	1.85	1.92	1.98	2.03		
12									-0.10	0.82	1.25	1.52	1.70	1.84	1.95	2.05	2.12	2.18		
13										0.07	1.00	1.43	1.70	1.89	2.04	2.15	2.25	2.32		
14											0.25	1.18	1.62	1.89	2.09	2.24	2.35	2.45		
15												0.43	1.37	1.81	2.09	2.29	2.44	2.55		
16													0.62	1.56	2.00	2.29	2.49	2.64		
17														0.81	1.75	2.20	2.49	2.69		
18															1.00	1.95	2.40	2.69		
19																1.20	2.15	2.60		
20																	1.40	2.35		

	FA-ExpectedSurplus																		
	S=3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
N=2	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	
3	0.46	0.46	0.46	0.46	0.46	0.46	0.46	0.46	0.46	0.46	0.46	0.46	0.46	0.46	0.46	0.46	0.46	0.46	
4	0.63	0.63	0.63	0.63	0.63	0.63	0.63	0.63	0.63	0.63	0.63	0.63	0.63	0.63	0.63	0.63	0.63	0.63	
5	0.79	0.79	0.79	0.79	0.79	0.79	0.79	0.79	0.79	0.79	0.79	0.79	0.79	0.79	0.79	0.79	0.79	0.79	
6	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	
7	1.13	1.13	1.13	1.13	1.13	1.13	1.13	1.13	1.13	1.13	1.13	1.13	1.13	1.13	1.13	1.13	1.13	1.13	
8	1.29	1.29	1.29	1.29	1.29	1.29	1.29	1.29	1.29	1.29	1.29	1.29	1.29	1.29	1.29	1.29	1.29	1.29	
9	1.46	1.46	1.46	1.46	1.46	1.46	1.46	1.46	1.46	1.46	1.46	1.46	1.46	1.46	1.46	1.46	1.46	1.46	
10	1.63	1.63	1.63	1.63	1.63	1.63	1.63	1.63	1.63	1.63	1.63	1.63	1.63	1.63	1.63	1.63	1.63	1.63	
11	1.79	1.79	1.79	1.79	1.79	1.79	1.79	1.79	1.79	1.79	1.79	1.79	1.79	1.79	1.79	1.79	1.79	1.79	
12	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96	
13	2.13	2.13	2.13	2.13	2.13	2.13	2.13	2.13	2.13	2.13	2.13	2.13	2.13	2.13	2.13	2.13	2.13	2.13	
14	2.29	2.29	2.29	2.29	2.29	2.29	2.29	2.29	2.29	2.29	2.29	2.29	2.29	2.29	2.29	2.29	2.29	2.29	
15	2.46	2.46	2.46	2.46	2.46	2.46	2.46	2.46	2.46	2.46	2.46	2.46	2.46	2.46	2.46	2.46	2.46	2.46	
16	2.63	2.63	2.63	2.63	2.63	2.63	2.63	2.63	2.63	2.63	2.63	2.63	2.63	2.63	2.63	2.63	2.63	2.63	
17	2.79	2.79	2.79	2.79	2.79	2.79	2.79	2.79	2.79	2.79	2.79	2.79	2.79	2.79	2.79	2.79	2.79	2.79	
18	2.96	2.96	2.96	2.96	2.96	2.96	2.96	2.96	2.96	2.96	2.96	2.96	2.96	2.96	2.96	2.96	2.96	2.96	
19	3.13	3.13	3.13	3.13	3.13	3.13	3.13	3.13	3.13	3.13	3.13	3.13	3.13	3.13	3.13	3.13	3.13	3.13	
20	3.29	3.29	3.29	3.29	3.29	3.29	3.29	3.29	3.29	3.29	3.29	3.29	3.29	3.29	3.29	3.29	3.29	3.29	

Figure 37: Table of Expected Surplus to Buyer from RA and FA Auction

								SRAFA											
	S-3	4	5	6	7	8	9	10	11	12	13	14	15	16	17				
N=2	15.00	7.57	5.86	5.09	4.65	4.36	4.16	4.01	3.90	3.81	3.73	3.67	3.62	3.57	3.54				
3	-9.00	19.00	8.00	6.00	5.15	4.68	4.38	4.18	4.02	3.91	3.81	3.74	3.67	3.62	3.58				
4	1.00	-7.67	23.00	8.33	6.11	5.21	4.71	4.40	4.19	4.03	3.91	3.82	3.74	3.68	3.62				
5	1.00	1.00	-7.00	27.00	8.60	6.20	5.25	4.74	4.42	4.20	4.04	3.92	3.82	3.74	3.68				
6	1.00	1.00	1.00	-6.60	31.00	8.82	6.27	5.29	4.76	4.43	4.21	4.05	3.92	3.83	3.75				
7	1.00	1.00	1.00	1.00	-6.33	35.00	9.00	6.33	5.32	4.78	4.44	4.22	4.05	3.93	3.83				
8	1.00	1.00	1.00	1.00	1.00	-6.14	38.00	9.15	6.38	5.34	4.79	4.45	4.22	4.06	3.93				
9	1.00	1.00	1.00	1.00	1.00	1.00	-6.00	43.00	9.29	6.43	5.36	4.81	4.46	4.23	4.06				
10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-5.89	47.00	9.40	6.47	5.38	4.82	4.47	4.24				
11	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-5.80	51.00	9.50	6.50	5.40	4.83	4.48				
12	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-5.73	55.00	9.59	6.53	5.42	4.84				
13	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-5.67	59.00	9.67	6.56	5.43				
14	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-5.62	63.00	9.74	6.58				
15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-5.57	67.00	9.80				
16	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-5.53	71.00				
17	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-5.50				
18	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00				
19	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00				
20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00				

								BRAFA											
	S-3	4	5	6	7	8	9	10	11	12	13	14	15	16	17				
N=2	-13.00	19.00	6.50	4.25	3.31	2.79	2.46	2.24	2.07	1.94	1.84	1.76	1.70	1.64	1.59				
3	-5.50	-46.00	10.25	5.24	3.75	3.04	2.62	2.35	2.15	2.01	1.89	1.80	1.73	1.67	1.62				
4	-2.85	-8.06	39.00	7.32	4.47	3.40	2.84	2.49	2.25	2.08	1.95	1.85	1.77	1.70	1.64				
5	0.00	-3.42	-13.50	14.91	5.84	3.96	3.14	2.68	2.38	2.17	2.02	1.90	1.81	1.74	1.67				
6	0.00	0.00	-4.16	-33.31	9.63	4.95	3.58	2.93	2.54	2.29	2.10	1.97	1.86	1.78	1.71				
7	0.00	0.00	0.00	-5.15	107.89	7.31	4.35	3.30	2.76	2.43	2.21	2.04	1.92	1.82	1.75				
8	0.00	0.00	0.00	0.00	-6.57	22.39	6.00	3.92	3.08	2.63	2.34	2.14	1.99	1.88	1.79				
9	0.00	0.00	0.00	0.00	0.00	-8.77	13.02	5.16	3.59	2.90	2.51	2.26	2.08	1.94	1.84				
10	0.00	0.00	0.00	0.00	0.00	0.00	-12.66	9.41	4.58	3.34	2.75	2.41	2.19	2.02	1.90				
11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-21.38	7.50	4.14	3.13	2.63	2.33	2.12	1.98				
12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-59.13	6.32	3.81	2.96	2.52	2.25	2.07				
13	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	92.03	5.51	3.54	2.82	2.43	2.19				
14	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	27.18	4.92	3.32	2.70	2.35				
15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	16.37	4.48	3.14	2.59				
16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	11.92	4.13	2.99				
17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	9.49	3.84				
18	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	7.96				
19	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00				
20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00				

Figure 38: Table of the Number of Buyers for Upper and Lower Bounds of Equilibrium Region

APPENDIX D

COMPARATIVE STATICS

In this appendix, we provide the benefit of an increase in the number of periods to the supplier or buyer for the PDSM RA, FA, and RFA auction designs.

Proposition 3.5.8(Preference of Sequence Length) The effect of increasing the number of auction periods is shown in Table 4 of Chapter 3.5.2.

Proof of Proposition 3.5.8

To determine the benefit of an increase in the number of periods to the supplier or buyer for the PDSM RA, FA, and RFA auction designs, we compute the expected value of the difference in expected return between the sequence with N periods and the sequence with $N + 1$ periods. The valuations are equal in expectation, (i.e., $E[v_{N+1}] = E[v_N]$).

PDSM Reverse Auction

The expected profit for a supplier participating in the N^{th} -to-last period of a PDSM reverse auction is

$$P_N = \frac{(1 - v_N)^S}{S} + \frac{N - 1}{S(S - N + 1)}$$

The benefit of each additional period to the expected supplier profit is as follows:

$$\begin{aligned} E[P_{N+1} - P_N] &= \int_0^1 \left[\frac{(1 - v_{N+1})^S}{S} dv_{N+1} \right] + \frac{(N + 1) - 1}{S(S - (N + 1) + 1)} \\ &\quad - \int_0^1 \left[\frac{(1 - v_N)^S}{S} dv_N \right] - \frac{N - 1}{S(S - N + 1)} \\ &= \frac{N}{S(S - N)} - \frac{N - 1}{S(S - N + 1)} \\ &= \frac{N(S - N + 1) - (N - 1)(S - N)}{S(S - N)(S - N + 1)} \\ &= \frac{NS - N^2 + N - [NS - S - N^2 + N]}{S(S - N)(S - N + 1)} \\ &= \frac{1}{(S - N)(S - N + 1)}. \end{aligned}$$

Note that the benefit of an additional period is positive. Therefore the expected profit increases with an increase in N .

The expected surplus for a buyer participating in the N^{th} -to-last period of a PDSM reverse auction is

$$D_N = \begin{cases} \frac{w_N - \frac{2}{S+1}}{B} + \sum_{p=1}^{N-1} \frac{\frac{1}{2} - \frac{2}{S-p+1}}{B}, & \text{when } N > 1 \\ \frac{w_N - \frac{2}{S+1}}{B}, & \text{when } N = 1. \end{cases}$$

We take the difference between a sequence with N periods and a sequence with $N + 1$ periods. The benefit from the expected surplus of an additional period is as follows:

$$\begin{aligned} E[D_{N+1} - D_N] &= \int_0^1 \left[\frac{w_{N+1} - \frac{2}{S+1}}{B} \right] dw_{N+1} + \sum_{p=1}^{N-1} \frac{\frac{1}{2} - \frac{2}{S-p+1}}{B} + \frac{\frac{1}{2} - \frac{2}{S-N+1}}{B} \\ &\quad - \left(\int_0^1 \left[\frac{w_N - \frac{2}{S+1}}{B} \right] dw_N + \sum_{p=1}^{N-1} \frac{\frac{1}{2} - \frac{2}{S-p+1}}{B} \right) \\ &= \frac{\frac{1}{2} - \frac{2}{S-N+1}}{B}. \end{aligned}$$

Note that the benefit of an additional period is positive for $S - N > 3$ and non-positive for $S - N \leq 3$. Therefore the expected surplus may increase or decrease with an increase in N depending on the difference between N and S .

PDSM Forward Auction

The expected profit for a supplier participating in the N^{th} -to-last period of a PDSM forward auction is

$$P_N = \frac{\frac{B-1}{B+1} - v_N}{S} + \sum_{p=0}^{N-2} \left(\prod_{q=0}^p \left(1 - \frac{1}{S-q} \right) \right) \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S-p-1}.$$

The benefit of an additional period is as follows:

$$\begin{aligned} E[P_{N+1} - P_N] &= \int_0^1 \left[\frac{\frac{B-1}{B+1} - v_{N+1}}{S} \right] dv_{N+1} + \sum_{p=0}^{N-2} \left(\prod_{q=0}^p \left(1 - \frac{1}{S-q} \right) \right) \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S-p-1} \\ &\quad + \left(\left(1 - \frac{1}{S} \right) \left(1 - \frac{1}{S-1} \right) \cdots \left(1 - \frac{1}{S-N+1} \right) \right) \frac{B-3}{2(B+1)(S-N)} \\ &\quad - \left(\int_0^1 \left[\frac{\frac{B-1}{B+1} - v_N}{S} \right] dv_N + \sum_{p=0}^{N-2} \left(\prod_{q=0}^p \left(1 - \frac{1}{S-q} \right) \right) \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S-p-1} \right) \\ &= \left(\left(\frac{S-1}{S} \right) \left(\frac{S-2}{S-1} \right) \left(\frac{S-3}{S-2} \right) \cdots \left(\frac{S-N+1}{S-N+2} \right) \left(\frac{S-N}{S-N+1} \right) \right) \frac{B-3}{2(B+1)(S-N+1)}. \end{aligned}$$

Note that the benefit of an additional period is negative for $B = 2$ and non-negative for $B \geq 3$.

The expected surplus for a buyer participating in the N^{th} -to-last period of a PDSM forward auction is

$$D_N = \frac{w_N^B}{B} + \frac{N-1}{B(B+1)}.$$

The benefit of an additional period to the expected buyer surplus is as follows:

$$\begin{aligned} E[D_{N+1} - D_N] &= \int_0^1 \left[\frac{w_{N+1}^B}{B} \right] dw_{N+1} + \frac{(N+1)-1}{B(B+1)} - \int_0^1 \left[\frac{w_N^B}{B} \right] dw_N - \frac{N-1}{B(B+1)} \\ &= \frac{N}{B(B+1)} - \frac{N-1}{B(B+1)} \\ &= \frac{1}{B(B+1)}. \end{aligned}$$

Note that the benefit of an additional period for the expected surplus is positive. Therefore the expected surplus increases with an increase in N .

PDSM Alternating Auction

The expected profit for a supplier participating in the N^{th} -to-last period of a PDSM alternating auction when N is odd is

$$\begin{aligned} P_N &= \frac{\frac{B-1}{B+1} - v_N}{S} + \Upsilon_N \\ \text{where } \Upsilon_N &= \sum_{p=0}^{\frac{N-3}{2}} \left(\prod_{q=0}^p \left(1 - \frac{1}{S-2q} \right) \right) \left[\frac{1}{(S-2p)(S-2p-1)} + \left(\frac{B-3}{2(B+1)} \right) \left(\frac{1}{S-2p-2} \right) \right], \end{aligned}$$

and when N is even is

$$\begin{aligned} P_N &= \frac{(1-v_N)^S}{S} + \frac{B-3}{2(B+1)(S-1)} + \Delta_N \\ \text{where } \Delta_N &= \sum_{p=1}^{\frac{N-2}{2}} \left(\prod_{q=1}^p \left(1 - \frac{1}{S-2q+1} \right) \right) \\ &\quad * \left[\frac{1}{(S-2p)(S-2p+1)} + \frac{B-3}{2(B+1)} * \frac{1}{S-2p-1} \right]. \end{aligned}$$

Because we have fixed the auction type in the last period as a forward auction, we increase the number of periods by two. An increase in the number of periods is given by the difference between P_{N+2} and P_N where P_{N+2} is the expected profit with $N+2$ periods.

When $N + 2$ is even, the benefit of this increase is as follows:

$$\begin{aligned}
E[P_{N+2} - P_N] &= \int_0^1 \left[\frac{(1 - v_{N+2})^S}{S} \right] dv_{N+2} + \frac{B-3}{2(B+1)(S-1)} + \Delta_N \\
&\quad + \left(\prod_{q=1}^{\frac{N}{2}} \left(1 - \frac{1}{S-2q+1} \right) \right) \left[\frac{1}{(S-N)(S-N+1)} + \frac{B-3}{2(B+1)} \frac{1}{S-N-1} \right] \\
&\quad - \left(\int_0^1 \left[\frac{(1 - v_N)^S}{S} \right] dv_N + \frac{B-3}{2(B+1)(S-1)} + \Delta_N \right) \\
&= \left(\prod_{q=1}^{\frac{N}{2}} \left(1 - \frac{1}{S-2q+1} \right) \right) \left[\frac{1}{(S-N)(S-N+1)} + \frac{B-3}{2(B+1)} \frac{1}{S-N-1} \right].
\end{aligned}$$

When $N + 2$ is odd, the benefit of an increase in the number of periods is similar (differing in the first product term) and is given as follows:

$$\begin{aligned}
E[P_{N+2} - P_N] &= \int_0^1 \left[\frac{\frac{B-1}{B+1} - v_{N+2}}{S} \right] dv_{N+2} + \Upsilon_N \\
&\quad + \left(\prod_{q=0}^{\frac{N-1}{2}} \left(1 - \frac{1}{S-2q} \right) \right) \left[\frac{1}{(S-N)(S-N+1)} + \frac{B-3}{2(B+1)} \frac{1}{S-N-1} \right] \\
&\quad - \left(\int_0^1 \left[\frac{\frac{B-1}{B+1} - v_N}{S} \right] dv_N + \Upsilon_N \right) \\
&= \left(\prod_{q=0}^{\frac{N-1}{2}} \left(1 - \frac{1}{S-2q} \right) \right) \left[\frac{1}{(S-N)(S-N+1)} + \frac{B-3}{2(B+1)} \frac{1}{S-N-1} \right].
\end{aligned}$$

Note that when $N + 2$ is even or odd an increase in the number of periods increases the expected profit when $B \geq 3$. When $B = 2$, an increase in the number of periods has a non-increasing affect on expected profit as follows:

$$\begin{aligned}
\frac{1}{(S-N)(S-N+1)} + \frac{B-3}{2(B+1)} \frac{1}{S-N-1} &= \frac{1}{(S-N)(S-N+1)} - \left(\frac{1}{6}\right) \frac{1}{S-N-1} \\
&= \frac{5S - 5N - S^2 - N^2 + 2SN - 6}{(S-N)(S-N-1)(S-N+1)}.
\end{aligned}$$

which is ≤ 0 when

$$\begin{aligned}
\frac{S^2 + N^2 - 2SN + 6}{5S - 5N} &\geq 1 \\
\frac{(S-N)(S-N) + 6}{5(S-N)} &\geq 1 \\
\frac{S-N}{5} + \frac{6}{5(S-N)} &\geq 1
\end{aligned}$$

which holds since $S - N \geq 1$.

The expected surplus for a buyer participating in the N^{th} -to-last period of a PDSM

alternating auction when N is odd is

$$D_N = \frac{w_N^B}{B} + \frac{N-1}{2B(B+1)} + \sum_{p=0}^{\frac{N-3}{2}} \frac{\frac{1}{2} - \frac{2}{S-2p}}{B},$$

and when N is even is

$$D_N = \frac{w_N - \frac{2}{S+1}}{B} + \frac{N}{2B(B+1)} + \sum_{p=1}^{\frac{N-2}{2}} \frac{\frac{1}{2} - \frac{2}{S-2p+1}}{B}.$$

Because we have fixed the auction type in the last period as a forward auction, we increase the number of periods by two. An increase in the number of periods is given by the expected value of the difference between D_{N+2} and D_N where D_{N+2} is the expected surplus with $N+2$ periods. When $N+2$ is even, the benefit of this increase is as follows:

$$\begin{aligned} E[D_{N+2} - D_N] &= \int_0^1 \left[\frac{w_{N+2} - \frac{2}{S+1}}{B} \right] dw_{N+2} + \frac{N+2}{2B(B+1)} + \sum_{p=1}^{\frac{N-2}{2}} \frac{\frac{1}{2} - \frac{2}{S-2p+1}}{B} + \frac{\frac{1}{2} - \frac{2}{S-N+1}}{B} \\ &\quad - \left(\int_0^1 \left[\frac{w_N - \frac{2}{S+1}}{B} \right] dw_N + \frac{N}{2B(B+1)} + \sum_{p=1}^{\frac{N-2}{2}} \frac{\frac{1}{2} - \frac{2}{S-2p+1}}{B} \right) \\ &= \frac{\frac{1}{2} - \frac{2}{S-N+1}}{B} - \frac{1}{(S-N+1)(S-N+2)} + \frac{1}{2B(B+1)} \end{aligned}$$

When $N+2$ is odd, the benefit of an increase is the same as shown in the following:

$$\begin{aligned} E[D_{N+2} - D_N] &= \int_0^1 \left[\frac{w_{N+2}^B}{B} \right] dw_{N+2} + \frac{N+1}{2B(B+1)} + \sum_{p=0}^{\frac{N-3}{2}} \frac{\frac{1}{2} - \frac{2}{S-2p}}{B} + \frac{\frac{1}{2} - \frac{2}{S-N+1}}{B} \\ &\quad - \left(\int_0^1 \left[\frac{w_N^B}{B} \right] dw_N + \frac{N-1}{2B(B+1)} + \sum_{p=0}^{\frac{N-3}{2}} \frac{\frac{1}{2} - \frac{2}{S-2p}}{B} \right) \\ &= \frac{\frac{1}{2} - \frac{2}{S-N+1}}{B} - \frac{1}{(S-N+1)(S-N+2)} + \frac{1}{2B(B+1)} \end{aligned}$$

Note that both when $N+2$ is even or odd an increase in the number of periods increases the expected surplus when $\frac{B+2}{2(B+1)} > \frac{2S-2N+4+B}{(S-N+1)(S-N+2)}$. ■

APPENDIX E

EXPECTED VALUE OF INFORMATION

In this appendix, we calculate the expected value of learning the valuation in the last period. We analyze two cases: when the decision to learn the last-period valuation is made before the second-to-last valuation v_2 is known, and when the decision is made after v_2 is known. We only consider these two models because the bid strategy of the buyer in the 2^{nd} -to-last period does not depend on his private valuation in the last period; whereas, the bid strategy of the supplier does.

Proposition 3.5.9(Expected Value of Information) For the reverse auction, the expected value to the supplier of knowing her valuation in the last period after choosing to participate in a two-period auction after (EVI_A) and before (EVI_B) learning her valuation in the 2^{nd} -to-last period are as follows:

When $v_2 > \frac{S-1}{S}$,

$$\begin{aligned} EVI_A(v_2) &= \left(\frac{(1-v_2)^S}{S(S-1)} \right) + \left(\frac{(1-Z)^S}{S(S-1)} \right) \\ &\quad + \left(\frac{1}{S} \left[\sum_{i=0}^S \left(1 - v_2 + \frac{1}{S(S-1)} \right)^i \binom{S}{i} \left[\frac{-(1-Z)^{(S-1)(S-i)+1}}{[(S-1)(S-i)+1](S-1)^{S-i}} \right] \right] \right) \\ &\quad - \left(\frac{(1-v_2)^S}{S} + \frac{1}{S(S-1)} \right). \end{aligned}$$

When $v_2 < \frac{1}{S(S-1)}$,

$$\begin{aligned} EVI_A(v_2) &= \left(\frac{1}{S-1} - v_2 - \frac{W}{S-1} + v_2 W \right) + \frac{1 - (1-W)^S}{S(S-1)} \\ &\quad + \left(\frac{1}{S} \left[\sum_{i=0}^S \left(1 - v_2 + \frac{1}{S(S-1)} \right)^i \binom{S}{i} \left[\frac{(1-W)^{(S-1)(S-i)+1} - 1}{[(S-1)(S-i)+1](S-1)^{S-i}} \right] \right] \right) \\ &\quad - \left(\frac{(1-v_2)^S}{S} + \frac{1}{S(S-1)} \right). \end{aligned}$$

When $\frac{1}{S(S-1)} \leq v_2 \leq \frac{S-1}{S}$,

$$\begin{aligned} EVI_A(v_2) &= \left(\frac{1}{S} \left[\sum_{i=0}^S \left(1 - v_2 + \frac{1}{S(S-1)} \right)^i \binom{S}{i} \left[\frac{-1}{[(S-1)(S-i)+1](S-1)^{S-i}} \right] \right] \right) \\ &\quad - \left(\frac{(1-v_2)^S}{S} \right). \end{aligned}$$

and

$$\begin{aligned}
EVI_B = & \left[\left(\frac{-(1-v_2)^{S+1}}{S(S-1)(S+1)} \right) \Big|_{\frac{S-1}{S}}^1 \right] + \left(\frac{[\frac{1}{S} + (1-v_2)(S-1)]^{\frac{S}{S-1}+1} - 1}{S(S-1)(\frac{S}{S-1}+1)} \frac{1}{S-1} \right) \Big|_{\frac{S-1}{S}}^1 \\
& + \int_{\frac{S-1}{S}}^1 \left(\frac{1}{S} \left[\sum_{i=0}^S (1-v_2 + \frac{1}{S(S-1)})^i \binom{S}{i} \left[\frac{[\frac{1}{S} + (1-v_2)(S-1)]^{(S-i)+\frac{1}{S-1}}}{[(S-1)(S-i)+1](S-1)^{S-i}} \right] \right] \right) dv_2 \\
& - \left(\frac{(1-v_2)^{S+1}}{S(S+1)} + \frac{v_2}{S(S-1)} \right) \Big|_{\frac{S-1}{S}}^1 \\
& + \int_{\frac{1}{S(S-1)}}^{\frac{S-1}{S}} \left[\left(\frac{1}{S-1} - v_2 - \frac{1 - [\frac{1}{S} - v_2(S-1)]^{\frac{1}{S-1}}}{S-1} + v_2 [1 - [\frac{1}{S} - v_2(S-1)]^{\frac{1}{S-1}}] \right) \right. \\
& + \left(\frac{1 - [\frac{1}{S} - v_2(S-1)]^{\frac{S}{S-1}}}{S(S-1)} \right) \\
& + \left(\frac{1}{S} \left[\sum_{i=0}^S (1-v_2 + \frac{1}{S(S-1)})^i \binom{S}{i} \left[\frac{[\frac{1}{S} - v_2(S-1)]^{(S-i)+\frac{1}{S-1}} - 1}{[(S-1)(S-i)+1](S-1)^{S-i}} \right] \right] \right) \\
& - \left(\frac{(1-v_2)^S}{S} + \frac{1}{S(S-1)} \right) \Big] f(v_2) dv_2 \\
& + \int_0^{\frac{1}{S(S-1)}} \left[\left(\frac{1}{S} \left[\sum_{i=0}^S (1-v_2 + \frac{1}{S(S-1)})^i \binom{S}{i} \left[\frac{-1}{[(S-1)(S-i)+1](S-1)^{S-i}} \right] \right] \right) \right. \\
& - \left(\frac{(1-v_2)^S}{S} \right) \Big] f(v_2) dv_2.
\end{aligned}$$

For the alternating auction, the expected value to the supplier of knowing her valuation in the last period after choosing to participate in a two-period auction after (EVI_A) and before (EVI_B) learning her valuation in the 2^{nd} -to-last period are as follows:

When $v_2 > 1 - \frac{1}{2(S-1)}$,

$$\begin{aligned}
EVI_A(v_2) = & \left(\frac{\frac{B-1}{B+1}X - \frac{X^2}{2}}{S-1} \right) \\
& + \left(\frac{\frac{B-1}{B+1} - \frac{1}{2}}{S-1} + \frac{(1-v_2 - \frac{\frac{B-1}{B+1}-1}{S-1} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \right. \\
& - \left(\frac{\frac{B-1}{B+1}X - \frac{X^2}{2}}{S-1} + \frac{(1-v_2 - \frac{\frac{B-1}{B+1}-X}{S-1} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \right) \\
& - \left[(1-v_2)^{S-1} \left(\frac{B-3}{2(B+1)(S-1)} \right) + \frac{(1-v_2)^S}{S} + \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S} \frac{(1-(1-v_2)^{S-1})}{S-1} \right].
\end{aligned}$$

When $v_2 < \frac{1}{2(S-1)}$,

$$\begin{aligned}
EVI_A(v_2) = & \left(\frac{B-1}{2(B+1)(S-1)} - v_2 + \frac{1}{S} \right) - \left(\frac{B-1}{2(B+1)(S-1)} Y - v_2 Y + \frac{Y}{S} \right) \\
& + \left(\frac{\frac{B-1}{B+1}Y - \frac{Y^2}{2}}{S-1} + \frac{(1-v_2 - \frac{\frac{B-1}{B+1}-Y}{S-1} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \right) \\
& - \left(\frac{(1-v_2 - \frac{\frac{B-1}{B+1}-Y}{S-1} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \right) \\
& - \left[(1-v_2)^{S-1} \left(\frac{B-3}{2(B+1)(S-1)} \right) + \frac{(1-v_2)^S}{S} + \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S} \frac{(1-(1-v_2)^{S-1})}{S-1} \right].
\end{aligned}$$

When $\frac{1}{2(S-1)} \leq v_2 \leq 1 - \frac{1}{2(S-1)}$,

$$\begin{aligned} EVI_A(v_2) = & \left(\frac{\frac{B-1}{B+1} - \frac{1}{2}}{S-1} + \frac{(1-v_2 - \frac{\frac{B-1}{B+1} - 1}{S-1} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \right) \\ & - \left(\frac{(1-v_2 - \frac{B-1}{(B+1)(S-1)} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \right) \\ & - \left[(1-v_2)^{S-1} \left(\frac{B-3}{2(B+1)(S-1)} \right) + \frac{(1-v_2)^S}{S} + \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S} \frac{(1-(1-v_2)^{S-1})}{S-1} \right]. \end{aligned}$$

and

$$\begin{aligned} EVI_B = & \int_{1-\frac{1}{2(S-1)}}^1 \left[\left(\frac{\frac{B-1}{B+1}X - \frac{X^2}{2}}{S-1} \right) \right. \\ & + \left(\frac{\frac{B-1}{B+1} - \frac{1}{2}}{S-1} + \frac{(1-v_2 - \frac{\frac{B-1}{B+1} - 1}{S-1} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \right. \\ & \left. \left. - \left(\frac{\frac{B-1}{B+1}X - \frac{X^2}{2}}{S-1} + \frac{(1-v_2 - \frac{\frac{B-1}{B+1} - X}{S-1} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \right) \right) \right] f(v_2) dv_2 \\ & - \left[(1-v_2)^{S-1} \left(\frac{B-3}{2(B+1)(S-1)} \right) + \frac{(1-v_2)^S}{S} + \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S} \frac{(1-(1-v_2)^{S-1})}{S-1} \right] \\ & + \int_0^{\frac{1}{2(S-1)}} \left[\left(\frac{B-1}{2(B+1)(S-1)} - v_2 + \frac{1}{S} \right) - \left(\frac{B-1}{2(B+1)(S-1)} Y - v_2 Y + \frac{Y}{S} \right) \right. \\ & + \left(\frac{\frac{B-1}{B+1}Y - \frac{Y^2}{2}}{S-1} + \frac{(1-v_2 - \frac{\frac{B-1}{B+1} - Y}{S-1} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \right. \\ & \left. \left. - \left(\frac{(1-v_2 - \frac{B-1}{(B+1)(S-1)} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \right) \right) \right. \\ & \left. - \left[(1-v_2)^{S-1} \left(\frac{B-3}{2(B+1)(S-1)} \right) + \frac{(1-v_2)^S}{S} + \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S} \frac{(1-(1-v_2)^{S-1})}{S-1} \right] \right] f(v_2) dv_2 \\ & + \int_{\frac{1}{2(S-1)}}^{1-\frac{1}{2(S-1)}} \left[\left(\frac{\frac{B-1}{B+1} - \frac{1}{2}}{S-1} + \frac{(1-v_2 - \frac{\frac{B-1}{B+1} - 1}{S-1} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \right) \right. \\ & \left. - \left(\frac{(1-v_2 - \frac{B-1}{(B+1)(S-1)} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \right) \right. \\ & \left. - \left[(1-v_2)^{S-1} \left(\frac{B-3}{2(B+1)(S-1)} \right) + \frac{(1-v_2)^S}{S} + \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S} \frac{(1-(1-v_2)^{S-1})}{S-1} \right] \right] f(v_2) dv_2. \end{aligned}$$

Proof of Proposition 3.5.8

Reverse Auction

We now prove that the expected profit to the supplier when the private valuation in the last period is fixed but unknown is as above. In equilibrium, the suppliers will use the optimal bid strategy derived in Proposition 3.4.1 (i.e., $b(v_2) = v_2 + \frac{1}{S(S-1)}$).

Using this optimal bid strategy of the suppliers, we solve for the expected profit,

$Q_2(v_2, v_1)$, for the supplier with a fixed but unknown valuation v_1 as follows:

$$\begin{aligned}
Q_2(v_2, v_1) &= \int_{v_2}^1 (b(v_{(2)}) - v_2)(S-1)(1-F(v_{(2)}))^{S-2} f(v_{(2)}) dv_{(2)} \\
&\quad + \int_0^{v_2} \frac{(1-v_1)^{S-1}}{S-1} (S-1)(1-F(v_{(2)}))^{S-2} f(v_{(2)}) dv_{(2)} \\
&= \int_{v_2}^1 (v_{(2)} + \frac{1}{S(S-1)} - v_2)(S-1)(1-v_{(2)})^{S-2} dv_{(2)} \\
&\quad + (1-v_1)^{S-1} \left(\frac{1-(1-v_2)^{S-1}}{S-1} \right).
\end{aligned}$$

We continue using integration-by-parts as follows:

$$\begin{aligned}
&\int_{v_2}^1 (v_{(2)} + \frac{1}{S(S-1)} - v_2)(S-1)(1-v_{(2)})^{S-2} dv_{(2)} \\
&= - \left[(1-v_{(2)})^{S-1} (v_{(2)} + \frac{1}{S(S-1)} - v_2) \right] \Big|_{v_2}^1 + \int_{v_2}^1 (1-v_{(2)})^{S-1} dv_{(2)} \\
&= \frac{(1-v_2)^S}{S} + \frac{(1-v_2)^{S-1}}{S(S-1)}.
\end{aligned}$$

Therefore,

$$Q_2(v_2, v_1) = \frac{(1-v_2)^S}{S} + \frac{(1-v_2)^{S-1}}{S(S-1)} + (1-v_1)^{S-1} \left(\frac{1-(1-v_2)^{S-1}}{S-1} \right).$$

We now derive the type that she will bid as for the reverse auction when v_1 is known as follows:

$$\begin{aligned}
P_2(v_2, v_1) &= \int_r^1 (b(v_{(2)}) - v_2)(S-1)(1-F(v_{(2)}))^{S-2} f(v_{(2)}) dv_{(2)} \\
&\quad + \int_0^r \left[\int_{v_1}^1 (x-v_1)(S-2)(1-F(x))^{S-3} f(x) dx \right] (S-1)(1-F(v_{(2)}))^{S-2} f(v_{(2)}) dv_{(2)} \\
&= \int_r^1 (v_{(2)} + \frac{1}{S(S-1)} - v_2)(S-1)(1-v_{(2)})^{S-2} dv_{(2)} \\
&\quad + \int_0^r \left[\int_{v_1}^1 (x-v_1)(S-2)(1-x)^{S-3} dx \right] (S-1)(1-v_{(2)})^{S-2} dv_{(2)} \\
&= \int_r^1 (v_{(2)} + \frac{1}{S(S-1)} - v_2)(S-1)(1-v_{(2)})^{S-2} dv_{(2)} \\
&\quad + \int_0^r (1-v_1)^{S-1} (1-v_{(2)})^{S-2} dv_{(2)}.
\end{aligned}$$

We obtain the first order conditions using Leibnitz's rule with respect to the type, r , in the 2^{nd} -to-last period where r is a function of v_2 and v_1 , we have

$$\frac{dP_2(v_2, v_1)}{dr} = - \left[\left(r + \frac{1}{S(S-1)} - v_2 \right) * (S-1)(1-r)^{S-2} \right] + (1-v_1)^{S-1} (1-r)^{S-2} \equiv 0$$

We solve for the type, r as follows:

$$\begin{aligned}
(r + \frac{1}{S(S-1)} - v_2)(S-1)(1-r)^{S-2} &= (1-v_1)^{S-1}(1-r)^{S-2} \\
(r + \frac{1}{S(S-1)} - v_2)(S-1) &= (1-v_1)^{S-1} \\
r + \frac{1}{S(S-1)} - v_2 &= \frac{(1-v_1)^{S-1}}{S-1} \\
r &= v_2 + \frac{(1-v_1)^{S-1}}{S-1} - \frac{1}{S(S-1)}.
\end{aligned}$$

We define $h(v_2, v_1) = v_2 + \frac{(1-v_1)^{S-1}}{S-1} - \frac{1}{S(S-1)}$. There are values of v_2 and v_1 that make $h(v_2, v_1) > 1$ or $h(v_2, v_1) < 0$ (i.e., $v_1 = 0$ and $v_2 = 1$ or $v_1 = 1$ and $v_2 = 0$). We now consider the expected profit for each of three cases: when $h(v_2, v_1) > 1$, when $h(v_2, v_1) < 0$, and when $0 < h(v_2, v_1) < 1$. We will use h in place of $h(v_2, v_1)$.

Limits of Integration We first consider the range of values of v_1 that correspond to $h(v_2, v_1) > 1$. The upper bound on v_1 is derived as follows:

$$\begin{aligned}
h(v_2, v_1) &> 1 \\
v_2 + \frac{(1-v_1)^{S-1}}{S-1} - \frac{1}{S(S-1)} &> 1 \\
v_2 + \frac{(1-v_1)^{S-1}}{S-1} &> 1 + \frac{1}{S(S-1)} \\
v_2(S-1) + (1-v_1)^{S-1} &> (S-1) + \frac{S-1}{S(S-1)} \\
(1-v_1)^{S-1} &> (S-1) + \frac{S-1}{S(S-1)} - v_2(S-1) \\
(1-v_1)^{S-1} &> \frac{1}{S} + (1-v_2)(S-1) \\
(1-v_1) &> [\frac{1}{S} + (1-v_2)(S-1)]^{\frac{1}{S-1}} \\
v_1 &< 1 - [\frac{1}{S} + (1-v_2)(S-1)]^{\frac{1}{S-1}} \\
v_1 &< Z,
\end{aligned}$$

where $Z = 1 - [\frac{1}{S} + (1-v_2)(S-1)]^{\frac{1}{S-1}}$. We now consider the values of v_2 for which $v_1 < Z$ is not possible, given that v_1 is bounded by $[0, 1]$. Specifically, when $Z \leq 0$, there is

no value of v_1 for which $v_1 < Z$ or for which $h > 1$. The inequality $Z \leq 0$ holds when

$$\begin{aligned}
Z &\leq 0 \\
1 - \left[\frac{1}{S} + (1 - v_2)(S - 1)\right]^{\frac{1}{S-1}} &\leq 0 \\
1 &\leq \left[\frac{1}{S} + (1 - v_2)(S - 1)\right]^{\frac{1}{S-1}} \\
\frac{1}{S} + (1 - v_2)(S - 1) &\geq 1 \\
(1 - v_2)(S - 1) &\geq 1 - \frac{1}{S} \\
(1 - v_2) &\geq \frac{1 - \frac{1}{S}}{S - 1} \\
(1 - v_2) &\geq \frac{S - 1}{S(S - 1)} \\
v_2 &\leq 1 - \frac{1}{S}.
\end{aligned}$$

Therefore, when $v_2 \leq 1 - \frac{1}{S}$ there is no v_1 for which $v_1 < Z$. So when $v_2 > 1 - \frac{1}{S}$, we are able to consider the expected profit where $h > 1$.

We next consider the values of v_1 that correspond to $h(v_2, v_1) < 0$. If $v_1 \geq Z$ then $h \leq 1$. The lower bound on v_1 is derived as follows:

$$\begin{aligned}
h(v_2, v_1) &< 0 \\
v_2 + \frac{(1 - v_1)^{S-1}}{S - 1} - \frac{1}{S(S - 1)} &< 0 \\
\frac{(1 - v_1)^{S-1}}{S - 1} &< \frac{1}{S(S - 1)} - v_2 \\
(1 - v_1)^{S-1} &< \frac{S - 1}{S(S - 1)} - v_2(S - 1) \\
(1 - v_1) &< \left[\frac{1}{S} - v_2(S - 1)\right]^{\frac{1}{S-1}} \\
v_1 &> 1 - \left[\frac{1}{S} - v_2(S - 1)\right]^{\frac{1}{S-1}} \\
v_1 &> W,
\end{aligned}$$

where $W = 1 - \left[\frac{1}{S} - v_2(S - 1)\right]^{\frac{1}{S-1}}$.

We now consider the values of v_2 for which $v_1 > W$ is not possible, given that v_1 is bounded by $[0, 1]$. Specifically, when $W \geq 1$, there is no value of v_1 for which $v_1 > W$ or

for which $h < 0$. The inequality $W \geq 1$ holds when

$$\begin{aligned}
W &\geq 1 \\
1 - \left[\frac{1}{S} - v_2(S-1)\right]^{\frac{1}{S-1}} &\geq 1 \\
0 &\geq \left[\frac{1}{S} - v_2(S-1)\right]^{\frac{1}{S-1}} \\
\frac{1}{S} - v_2(S-1) &\leq 0 \\
\frac{1}{S} &\leq v_2(S-1) \\
\frac{1}{S(S-1)} &\leq v_2
\end{aligned}$$

Therefore, when $v_2 \geq \frac{1}{S(S-1)}$, there is no v_1 for which $v_1 > W$. So when $v_2 < \frac{1}{S(S-1)}$ we are able to consider the expected profit when $h < 0$.

When we compare the bounds on v_2 we have

$$\begin{aligned}
&\frac{S-1}{S} - \frac{1}{S(S-1)} \\
&= \frac{(S-1)^2 - 1}{S(S-1)} > 0 \text{ when } S > 2
\end{aligned}$$

Therefore, $\frac{S-1}{S} > \frac{1}{S(S-1)}$ and the ranges on v_2 for $h < 0$ and $h > 1$ do not intersect.

We also consider the limits of integration when $0 \leq h \leq 1$. These limits are $Z \leq v_1 \leq W$ with the corresponding bounds for v_2 as $\frac{1}{S(S-1)} \leq v_2 \leq \frac{S-1}{S}$.

With these limits of integration, we obtain the expected value of information to the supplier who knows what her private valuation will be in the last period for the cases when $h(v_2, v_1) > 1$, $h(v_2, v_1) < 0$, and $0 < h(v_2, v_1) < 1$.

Case 1: In this case, $h(v_2, v_1) > 1$ so the strategic supplier will lose the first auction and receive her expected profit in the last period. If $v_2 > \frac{S-1}{S}$, the expected value of learning v_1 in the 2^{nd} -to-last period after v_2 is known ($EVI_A(v_2)$) is as follows:

$$\begin{aligned}
EVI_A(v_2) &= \int_0^Z \left(\frac{(1-v_1)^{S-1}}{S-1} \right) f(v_1) dv_1 \\
&+ \int_Z^1 \left(\frac{(1-v_1)^{S-1}}{S-1} + \frac{(1-v_2 - \frac{(1-v_1)^{S-1}}{S-1} + \frac{1}{S(S-1)})^S}{S} \right) f(v_1) dv_1 \\
&- \int_0^1 Q_2(v_2, v_1) f(v_1) dv_1.
\end{aligned}$$

where the first integrand is the expected profit in the last period and follows from Proposition 3.4.1 for P_1 . The third integrand is the expected profit to the supplier when she does not know v_1 until the last period so it is integrated over all possible values of v_1 . The second integrand is the expected profit when $0 \leq h \leq 1$ because although $v_2 > \frac{S-1}{S}$, it is possible that $v_1 > Z$. When this occurs, the supplier may win in either period. The integrand is derived as follows:

$$\begin{aligned}
& \int_h^1 (b(v_{(2)}) - v_2)(S-1)(1-F(v_{(2)}))^{S-2} f(v_{(2)}) dv_{(2)} \\
& + \int_0^h \left[\int_{v_1}^1 (x-v_1)(S-2)(1-F(x))^{S-3} f(x) dx \right] \\
& * (S-1)(1-F(v_{(2)}))^{S-2} f(v_{(2)}) dv_{(2)} \\
= & \int_h^1 \left(v_{(2)} + \frac{1}{S(S-1)} - v_2 \right) (S-1)(1-v_{(2)})^{S-2} dv_{(2)} \\
& + \int_0^h \left[\int_{v_1}^1 (x-v_1)(S-2)(1-x)^{S-3} dx \right] (S-1)(1-v_{(2)})^{S-2} dv_{(2)} \\
= & - \left[(1-v_{(2)})^{S-1} \left(v_{(2)} + \frac{1}{S(S-1)} - v_2 \right) \right]_h^1 + \int_h^1 (1-v_{(2)})^{S-1} dv_{(2)} \\
& + \int_0^h \left(- \left[(1-x)^{S-2} (x-v_1) \right]_{v_1}^1 + \int_{v_1}^1 (1-v_{(2)})^{S-2} dv_{(2)} \right) \\
& * (S-1)(1-v_{(2)})^{S-2} dv_{(2)} \\
= & (1-v_1)^{S-1} \left(\frac{(1-h)^{S-1}}{S-1} \right) \\
& + \frac{(1-h)^S}{S} + \int_0^h (1-v_1)^{S-1} (1-v_{(2)})^{S-2} dv_{(2)} \\
= & (1-v_1)^{S-1} \left(\frac{(1-h)^{S-1}}{S-1} \right) \\
& + \frac{(1-h)^S}{S} + (1-v_1)^{S-1} \left(\frac{1-(1-h)^{S-1}}{S-1} \right) \\
= & \frac{(1-v_1)^{S-1}}{S-1} + \frac{(1-h)^S}{S} \\
= & \frac{(1-v_1)^{S-1}}{S-1} + \frac{(1-v_2 - \frac{(1-v_1)^{S-1}}{S-1} + \frac{1}{S(S-1)})^S}{S}.
\end{aligned} \tag{38}$$

$EVI_A(v_2)$ is simplified as follows:

$$\begin{aligned}
EVI_A(v_2) &= \int_0^Z \left(\frac{(1-v_1)^{S-1}}{S-1} \right) f(v_1) dv_1 \\
&\quad + \int_Z^1 \left(\frac{(1-v_1)^{S-1}}{S-1} + \frac{(1-v_2 - \frac{(1-v_1)^{S-1}}{S-1} + \frac{1}{S(S-1)})^S}{S} \right) f(v_1) dv_1 \\
&\quad - \int_0^1 Q_2(v_2, v_1) f(v_1) dv_1 \\
&= \left(\frac{-(1-v_1)^S}{S(S-1)} \right) \Big|_0^Z + \left(\frac{-(1-v_1)^S}{S(S-1)} \right) \Big|_Z^1 \\
&\quad + \left(\frac{1}{S} \left[\sum_{i=0}^S (1-v_2 + \frac{1}{S(S-1)})^i \binom{S}{i} \left[\frac{(1-v_1)^{(S-1)(S-i)+1}}{[(S-1)(S-i)+1](S-1)^{S-i}} \right] \Big|_Z^1 \right] \right) \\
&\quad - \left(\left[\frac{(1-v_2)^S v_1}{S} + \frac{(1-v_2)^{S-1} v_1}{S(S-1)} + \frac{-(1-v_1)^S}{S(S-1)} - \frac{-(1-v_1)^S (1-v_2)^{S-1}}{S(S-1)} \right] \Big|_0^1 \right) \\
&= \left(\frac{(1-v_2)^S}{S} - \frac{2}{S(S-1)} \right) \\
&\quad + \left(\frac{1}{S} \left[\sum_{i=0}^S (1-v_2 + \frac{1}{S(S-1)})^i \binom{S}{i} \left[\frac{-(1-v_1)^{(S-1)(S-i)+1}}{[(S-1)(S-i)+1](S-1)^{S-i}} \right] \right] \right),
\end{aligned} \tag{39}$$

where the term on line (40) is derived using the integral of the binomial expansion of the second term on line (39) as follows:

$$\begin{aligned}
&\frac{1}{S} \left(\int_Z^1 (1-v_2 + \frac{1}{S(S-1)} + \frac{-(1-v_1)^{S-1}}{S-1})^S f(v_1) dv_1 \right) \\
&= \frac{1}{S} \left(\int_Z^1 \left[\sum_{i=0}^S (1-v_2 + \frac{1}{S(S-1)})^i \left(\frac{-(1-v_1)^{S-1}}{S-1} \right)^{S-i} \binom{S}{i} \right] dv_1 \right) \\
&= \frac{1}{S} \left(\sum_{i=0}^S \left[\int_Z^1 \left[(1-v_2 + \frac{1}{S(S-1)})^i \left(\frac{-(1-v_1)^{S-1}}{S-1} \right)^{S-i} \binom{S}{i} \right] dv_1 \right] \right) \\
&= \frac{1}{S} \left(\sum_{i=0}^S \left[\int_Z^1 \left[(1-v_2 + \frac{1}{S(S-1)})^i \left(\frac{-(1-v_1)^{(S-1)(S-i)+1}}{(S-1)^{S-i}} \right) \binom{S}{i} \right] dv_1 \right] \right) \\
&= \frac{1}{S} \left(\sum_{i=0}^S \left[(1-v_2 + \frac{1}{S(S-1)})^i \left(\frac{(1-v_1)^{(S-1)(S-i)+1}}{((S-1)(S-i)+1)(S-1)^{S-i}} \right) \binom{S}{i} \Big|_Z^1 \right] \right).
\end{aligned}$$

Case 2: In this case $h(v_2, v_1) < 0$, so the strategic supplier will win the first auction and receive her expected profit which depends on the bid of the most competitive bidder. She will not participate in the last period. If $v_2 < \frac{1}{S(S-1)}$, the expected value of learning v_1 in the 2^{nd} -to-last period after v_2 is known ($EVI_A(v_2)$) is as follows:

$$\begin{aligned}
EVI_A(v_2) &= \int_W^1 \left(\frac{1}{S-1} - v_2 \right) f(v_1) dv_1 \\
&\quad + \int_0^W \left(\frac{(1-v_1)^{S-1}}{S-1} + \frac{(1-v_2 - \frac{(1-v_1)^{S-1}}{S-1} + \frac{1}{S(S-1)})^S}{S} \right) f(v_1) dv_1 \\
&\quad - \int_0^1 Q_2(v_2, v_1) f(v_1) dv_1.
\end{aligned}$$

The second integrand is for the case when $v_1 < W$ which corresponds to $0 < h(v_2, v_1) < 1$ when the supplier could win in either period. The derivation is in (38). The third integrand

is the expected profit when v_1 is not known before the last period. The first integrand is the expected profit when $h < 0$ and is derived as follows:

$$\begin{aligned}
& \int_0^1 (b(v_{(2)}) - v_2)(S-1)(1-v_{(2)})^{S-2} dv_{(2)} \Big] f(v_2) dv_2 \\
&= - \left[(1-v_{(2)})^{S-1} (v_{(2)} + \frac{1}{S(S-1)} - v_2) \right] \Big|_0^1 + \int_0^1 (1-v_{(2)})^{S-1} dv_{(2)} \\
&= \frac{1}{S(S-1)} - v_2 + \frac{-(1-v_{(2)})^S}{S} \Big|_0^1 \\
&= \frac{1}{S(S-1)} - v_2 + \frac{1}{S} \\
&= \frac{1}{S-1} - v_2.
\end{aligned}$$

$EVI_A(v_2)$ is simplified as follows:

$$\begin{aligned}
EVI_A(v_2) &= \int_W^1 \left(\frac{1}{S-1} - v_2 \right) f(v_1) dv_1 \\
&+ \int_0^W \left(\frac{(1-v_1)^{S-1}}{S-1} + \frac{(1-v_2 - \frac{(1-v_1)^{S-1}}{S-1} + \frac{1}{S(S-1)})^S}{S} \right) f(v_1) dv_1 \quad (41) \\
&- \int_0^1 Q_2(v_2, v_1) f(v_1) dv_1
\end{aligned}$$

$$\begin{aligned}
&= \left(\frac{v_1}{S-1} - v_2 v_1 \right) \Big|_W^1 + \left(\frac{-(1-v_1)^S}{S(S-1)} \right) \Big|_0^W \\
&+ \left(\frac{1}{S} \left[\sum_{i=0}^S (1-v_2 + \frac{1}{S(S-1)})^i \binom{S}{i} \left[\frac{(1-v_1)^{(S-1)(S-i)+1}}{[(S-1)(S-i)+1](S-1)^{S-i}} \right] \Big|_0^W \right] \right) \quad (42) \\
&- \left(\left[\frac{(1-v_2)^S v_1}{S} + \frac{(1-v_2)^{S-1} v_1}{S(S-1)} + \frac{-(1-v_1)^S}{S(S-1)} - \frac{-(1-v_1)^S (1-v_2)^{S-1}}{S(S-1)} \right] \Big|_0^1 \right) \\
&= \left(\frac{1}{S-1} - v_2 - \frac{W}{S-1} + v_2 W \right) + \frac{1 - (1-W)^S}{S(S-1)} \\
&+ \left(\frac{1}{S} \left[\sum_{i=0}^S (1-v_2 + \frac{1}{S(S-1)})^i \binom{S}{i} \left[\frac{(1-W)^{(S-1)(S-i)+1} - 1}{[(S-1)(S-i)+1](S-1)^{S-i}} \right] \right] \right) \\
&- \left(\frac{(1-v_2)^S}{S} + \frac{1}{S(S-1)} \right),
\end{aligned}$$

where the term on line (42) is derived using the integral of the binomial expansion of the second term of line (41) as follows:

$$\begin{aligned}
& \frac{1}{S} \left(\int_0^W (1-v_2 + \frac{1}{S(S-1)} + \frac{-(1-v_1)^{S-1}}{S-1})^S f(v_1) dv_1 \right) \\
&= \frac{1}{S} \left(\int_0^W \left[\sum_{i=0}^S (1-v_2 + \frac{1}{S(S-1)})^i \left(\frac{-(1-v_1)^{S-1}}{S-1} \right)^{S-i} \binom{S}{i} \right] dv_1 \right) \\
&= \frac{1}{S} \left(\sum_{i=0}^S \left[\int_0^W \left[(1-v_2 + \frac{1}{S(S-1)})^i \left(\frac{-(1-v_1)^{S-1}}{S-1} \right)^{S-i} \binom{S}{i} \right] dv_1 \right] \right) \\
&= \frac{1}{S} \left(\sum_{i=0}^S \left[\int_0^W \left[(1-v_2 + \frac{1}{S(S-1)})^i \left(\frac{-(1-v_1)^{(S-1)(S-i)+1}}{(S-1)^{S-i}} \right) \binom{S}{i} \right] dv_1 \right] \right) \\
&= \frac{1}{S} \left(\sum_{i=0}^S \left[(1-v_2 + \frac{1}{S(S-1)})^i \left(\frac{(1-v_1)^{(S-1)(S-i)+1}}{((S-1)(S-i)+1)(S-1)^{S-i}} \right) \binom{S}{i} \Big|_0^W \right] \right).
\end{aligned}$$

Case 3: In this case $0 < h(v_2, v_1) < 1$, so the strategic supplier has a positive probability of winning either the first or second auction. If $\frac{1}{S(S-1)} \leq v_2 \leq \frac{S-1}{S}$, the expected value of learning v_1 in the 2^{nd} -to-last period after v_2 is known ($EVI_A(v_2)$) is as follows:

$$EVI_A(v_2) = \int_0^1 \left(\frac{(1-v_1)^{S-1}}{S-1} + \frac{(1-v_2 - \frac{(1-v_1)^{S-1}}{S-1} + \frac{1}{S(S-1)})^S}{S} \right) f(v_1) dv_1 - \int_0^1 Q_2(v_2, v_1) f(v_1) dv_1.$$

The first integrand is the expected profit when $0 \leq h \leq 1$ because although $\frac{1}{S(S-1)} \leq v_2 \leq \frac{S-1}{S}$, it is possible that $Z < v_1 < W$. When this occurs, the supplier may win in either period. The derivation of this integrand is given in (38). The second integrand is the expected profit when v_1 is not known before the last period.

$EVI_A(v_2)$ is simplified as follows:

$$\begin{aligned} EVI_A(v_2) &= \int_0^1 \left(\frac{(1-v_1)^{S-1}}{S-1} + \frac{(1-v_2 - \frac{(1-v_1)^{S-1}}{S-1} + \frac{1}{S(S-1)})^S}{S} \right) f(v_1) dv_1 \\ &\quad - \int_0^1 Q_2(v_2, v_1) f(v_1) dv_1 \\ &= \left(\frac{-(1-v_1)^S}{S(S-1)} \right) \Big|_0^1 \\ &\quad + \left(\frac{1}{S} \left[\sum_{i=0}^S (1-v_2 + \frac{1}{S(S-1)})^i \binom{S}{i} \left[\frac{(1-v_1)^{(S-1)(S-i)+1}}{[(S-1)(S-i)+1](S-1)^{S-i}} \right] \Big|_0^1 \right] \right) \\ &\quad - \left(\left[\frac{(1-v_2)^S v_1}{S} + \frac{(1-v_2)^{S-1} v_1}{S(S-1)} + \frac{-(1-v_1)^S}{S(S-1)} - \frac{-(1-v_1)^S (1-v_2)^{S-1}}{S(S-1)} \right] \Big|_0^1 \right) \\ &= \left(\frac{1}{S(S-1)} \right) \\ &\quad + \left(\frac{1}{S} \left[\sum_{i=0}^S (1-v_2 + \frac{1}{S(S-1)})^i \binom{S}{i} \left[\frac{-1}{[(S-1)(S-i)+1](S-1)^{S-i}} \right] \right] \right) \\ &\quad - \left(\frac{(1-v_2)^S}{S} + \frac{1}{S(S-1)} \right) \\ &= \left(\frac{1}{S} \left[\sum_{i=0}^S (1-v_2 + \frac{1}{S(S-1)})^i \binom{S}{i} \left[\frac{-1}{[(S-1)(S-i)+1](S-1)^{S-i}} \right] \right] \right) \\ &\quad - \left(\frac{(1-v_2)^S}{S} \right), \end{aligned} \tag{44}$$

where the term on line(44) is derived using the integral of the binomial expansion of the

second term of line (43) as follows:

$$\begin{aligned}
& \frac{1}{S} \left(\int_0^1 \left(1 - v_2 + \frac{1}{S(S-1)} + \frac{-(1-v_1)^{S-1}}{S-1} \right)^S f(v_1) dv_1 \right) \\
&= \frac{1}{S} \left(\int_0^1 \left[\sum_{i=0}^S \left(1 - v_2 + \frac{1}{S(S-1)} \right)^i \left(\frac{-(1-v_1)^{S-1}}{S-1} \right)^{S-i} \binom{S}{i} \right] dv_1 \right) \\
&= \frac{1}{S} \left(\sum_{i=0}^S \left[\int_0^1 \left[\left(1 - v_2 + \frac{1}{S(S-1)} \right)^i \left(\frac{-(1-v_1)^{S-1}}{S-1} \right)^{S-i} \binom{S}{i} \right] dv_1 \right] \right) \\
&= \frac{1}{S} \left(\sum_{i=0}^S \left[\int_0^1 \left[\left(1 - v_2 + \frac{1}{S(S-1)} \right)^i \left(\frac{-(1-v_1)^{(S-1)(S-i)}}{(S-1)^{S-i}} \right) \binom{S}{i} \right] dv_1 \right] \right) \\
&= \frac{1}{S} \left(\sum_{i=0}^S \left[\left(1 - v_2 + \frac{1}{S(S-1)} \right)^i \left(\frac{(1-v_1)^{(S-1)(S-i)+1}}{((S-1)(S-i)+1)(S-1)^{S-i}} \right) \binom{S}{i} \right]_0^1 \right).
\end{aligned}$$

To determine the amount that a supplier would be willing to pay to learn v_1 in the 2nd-to-last period before v_2 is known, (EVI_B), we integrate the expected value of information

in Cases 1-3 with respect to v_2 as well then sum them to obtain the following:

$$\begin{aligned}
EVI_B &= \int_{\frac{S-1}{S}}^1 \left[\int_0^Z \left(\frac{(1-v_1)^{S-1}}{S-1} - \frac{2}{S(S-1)} \right) f(v_1) dv_1 \right. \\
&\quad + \int_Z^1 \left(\frac{(1-v_1)^{S-1}}{S-1} + \frac{(1-v_2 - \frac{(1-v_1)^{S-1}}{S-1} + \frac{1}{S(S-1)})^S}{S} \right) f(v_1) dv_1 \\
&\quad \left. - \int_0^1 Q_2(v_2, v_1) f(v_1) dv_1 \right] f(v_2) dv_2 \\
&\quad + \int_{\frac{S-1}{S(S-1)}}^{\frac{S-1}{S}} \left[\int_0^1 \left(\frac{(1-v_1)^{S-1}}{S-1} + \frac{(1-v_2 - \frac{(1-v_1)^{S-1}}{S-1} + \frac{1}{S(S-1)})^S}{S} \right) f(v_1) dv_1 \right. \\
&\quad \left. - \int_0^1 Q_2(v_2, v_1) f(v_1) dv_1 \right] f(v_2) dv_2 \\
&\quad + \int_0^{\frac{1}{S(S-1)}} \left[\int_W^1 \left(\frac{1}{S(S-1)} - v_2 + \frac{1}{S} \right) f(v_1) dv_1 \right. \\
&\quad + \int_0^W \left(\frac{(1-v_1)^{S-1}}{S-1} + \frac{(1-v_2 - \frac{(1-v_1)^{S-1}}{S-1} + \frac{1}{S(S-1)})^S}{S} \right) f(v_1) dv_1 \\
&\quad \left. - \int_0^1 Q_2(v_2, v_1) f(v_1) dv_1 \right] f(v_2) dv_2 \\
&= \int_{\frac{S-1}{S}}^1 \left[\left(\frac{(1-v_2)^S}{S} - \frac{2}{S(S-1)} \right) \right. \\
&\quad + \left(\frac{1}{S} \left[\sum_{i=0}^S \left(1-v_2 + \frac{1}{S(S-1)} \right)^i \binom{S}{i} \left[\frac{-(1-Z)^{(S-1)(S-i)+1}}{[(S-1)(S-i)+1](S-1)^{S-i}} \right] \right] \right) f(v_2) dv_2 \\
&\quad + \int_{\frac{S-1}{S(S-1)}}^{\frac{S-1}{S}} \left[\left(\frac{1}{S-1} - v_2 - \frac{W}{S-1} + v_2 W \right) + \frac{1 - (1-W)^S}{S(S-1)} \right. \\
&\quad + \left(\frac{1}{S} \left[\sum_{i=0}^S \left(1-v_2 + \frac{1}{S(S-1)} \right)^i \binom{S}{i} \left[\frac{(1-W)^{(S-1)(S-i)+1} - 1}{[(S-1)(S-i)+1](S-1)^{S-i}} \right] \right] \right) \\
&\quad \left. - \left(\frac{(1-v_2)^S}{S} + \frac{1}{S(S-1)} \right) \right] f(v_2) dv_2 \\
&\quad + \int_0^{\frac{1}{S(S-1)}} \left[\left(\frac{1}{S} \left[\sum_{i=0}^S \left(1-v_2 + \frac{1}{S(S-1)} \right)^i \binom{S}{i} \left[\frac{-1}{[(S-1)(S-i)+1](S-1)^{S-i}} \right] \right] \right) \right. \\
&\quad \left. - \left(\frac{(1-v_2)^S}{S} \right) \right] f(v_2) dv_2,
\end{aligned}$$

We continue after substituting the values of Z and W where $Z = 1 - [\frac{1}{S} + (1-v_2)(S-1)]^{\frac{1}{S-1}}$

and where $W = 1 - [\frac{1}{S} - v_2(S-1)]^{\frac{1}{S-1}}$.

$$\begin{aligned}
EVI_B &= \int_{\frac{S-1}{S}}^1 \left[\left(\frac{(1-v_2)^S}{S} - \frac{2}{S(S-1)} \right) \right. \\
&\quad + \left(\frac{1}{S} \left[\sum_{i=0}^S (1-v_2 + \frac{1}{S(S-1)})^i \binom{S}{i} \left[\frac{[\frac{1}{S} + (1-v_2)(S-1)]^{(S-i)+\frac{1}{S-1}}}{[(S-1)(S-i)+1](1-S)^{S-i}} \right] \right) \right] f(v_2) dv_2 \\
&\quad + \int_{\frac{1}{S(S-1)}}^{\frac{S-1}{S}} \left[\left(\frac{1}{S-1} - v_2 - \frac{1 - [\frac{1}{S} - v_2(S-1)]^{\frac{1}{S-1}}}{S-1} + v_2 [1 - [\frac{1}{S} - v_2(S-1)]^{\frac{1}{S-1}}] \right) \right. \\
&\quad + \left(\frac{1 - [\frac{1}{S} - v_2(S-1)]^{\frac{S}{S-1}}}{S(S-1)} \right) \\
&\quad + \left(\frac{1}{S} \left[\sum_{i=0}^S (1-v_2 + \frac{1}{S(S-1)})^i \binom{S}{i} \left[\frac{[1 - \frac{1}{S} - v_2(S-1)]^{(S-i)+\frac{1}{S-1}}}{[(S-1)(S-i)+1](1-S)^{S-i}} \right] \right) \right] \\
&\quad - \left(\frac{(1-v_2)^S}{S} + \frac{1}{S(S-1)} \right) \left. \right] f(v_2) dv_2 \\
&\quad + \int_0^{\frac{1}{S(S-1)}} \left[\left(\frac{1}{S} \left[\sum_{i=0}^S (1-v_2 + \frac{1}{S(S-1)})^i \binom{S}{i} \left[\frac{1}{[(S-1)(S-i)+1](1-S)^{S-i}} \right] \right) \right] \right. \\
&\quad - \left(\frac{(1-v_2)^S}{S} \right) \left. \right] f(v_2) dv_2 \\
&= \left[\left(\frac{-(1-v_2)^{S+1}}{S(S+1)} - \frac{2v_2}{S(S-1)} \right) \Big|_{\frac{S-1}{S}}^1 \right] + \left(\frac{1}{S^3} \left[\sum_{i=0}^S \binom{S}{i} \frac{1}{[(S-1)(S-i)+1](1-S)^{S-i}} \right. \right. \\
&\quad * \left[e^{((S-i)+\frac{1}{S-1})(\text{Log}[1-v_2+\frac{1}{S(S-1)}]+\text{Log}[\frac{1}{S}+(1-v_2)(S-1)])} \right. \\
&\quad \left. \left. * (1-v_2 + \frac{1}{S(S-1)})^{S+\frac{1}{S-1}} (1-S)(1-v_2) - 1 \right] \right] \Big|_{\frac{S-1}{S}}^1 \Bigg) \\
&\quad + \left[\left(\left(-\frac{v_2^2}{2} - \frac{[\frac{1}{S} - v_2(S-1)]^{\frac{S}{S-1}}}{S(S-1)} + v_2 [-1 - [\frac{1}{S} - v_2(S-1)]^{\frac{S}{S-1}}] \right. \right. \right. \\
&\quad \left. \left. + \frac{v_2 - [\frac{1}{S} - v_2(S-1)]^{\frac{2S-1}{S-1}}}{S(2S-1)} \right) \Big|_{\frac{1}{S(S-1)}}^{\frac{S-1}{S}} \right) - \left(\frac{[v_2 - [\frac{1}{S} - v_2(S-1)]^{\frac{2S-1}{S-1}}]}{S(2S-1)} \Big|_{\frac{1}{S(S-1)}}^{\frac{S-1}{S}} \right) \\
&\quad + \left(\frac{1}{S} \left[\sum_{i=0}^S \binom{S}{i} \frac{1}{[(S-1)(S-i)+1](1-S)^{S-i}} \right. \right. \\
&\quad * \left. \int_{\frac{1}{S(S-1)}}^{\frac{S-1}{S}} (1-v_2 + \frac{1}{S(S-1)})^i \left[1 - \frac{1}{S} - v_2(S-1) \right]^{(S-i)+\frac{1}{S-1}} dv_2 \right] \Bigg) \\
&\quad - \left(\frac{-(1-v_2)^{S+1}}{S(S+1)} + \frac{v_2}{S(S-1)} \right) \Big|_{\frac{1}{S(S-1)}}^{\frac{S-1}{S}} \Bigg) \\
&\quad * \left(\frac{(1-v_2)^{S+1}}{S(S+1)} \Big|_0^{\frac{1}{S(S-1)}} \right).
\end{aligned}$$

We simplify as follows:

$$\begin{aligned}
EVI_B = & \left[\left(\frac{1}{S^{S+2}(S-1)(S+1)} - \frac{2}{S(S-1)} + \frac{2}{S^2} \right) + \left(\frac{[(\frac{2}{S})^{\frac{2S-1}{S-1}} - (\frac{1}{S})^{\frac{2S-1}{S-1}}]}{\frac{S(2S-1)}{S-1}} \right) \right. \\
& + \left(\frac{1}{S^3} \left[\sum_{i=0}^S \binom{S}{i} \frac{1}{[(S-1)(S-i)+1](1-S)^{S-i}} \right. \right. \\
& * \left[e^{((S-i)+\frac{1}{S-1})(\text{Log}[\frac{1}{S(S-1)}] + \text{Log}[\frac{1}{S}])} - 1 \right] - \left[e^{((S-i)+\frac{1}{S-1})\text{Log}[\frac{1}{S-1}]} \left(\frac{1-2S}{S} \right) \right] \left. \right] \left. \right) \\
& - \left(\frac{1}{S(S-1)} + \frac{1}{S^{S+2}(S+1)} - \frac{1}{S^2} \right) \left. \right] \\
& + \left[\left(-\frac{(S-1)^2}{2S^2} - \frac{[\frac{1}{S} - \frac{(S-1)^2}{S}]^{\frac{S}{S-1}}}{S(S-1)} + \frac{S-1}{S} [-1 - [\frac{1}{S} - \frac{(S-1)^2}{S}]^{\frac{S}{S-1}}] \right) \right. \\
& - \frac{1}{2S^2(S-1)^2} + \frac{1}{S(S-1)} \left. \right) \\
& + \left(\frac{1}{S} \left[\sum_{i=0}^S \binom{S}{i} \frac{1}{[(S-1)(S-i)+1](1-S)^{S-i}} \right. \right. \\
& * \left. \int_{\frac{1}{S(S-1)}}^{\frac{S-1}{S}} (1-v_2 + \frac{1}{S(S-1)})^i \left[1 - \frac{1}{S} - v_2(S-1) \right]^{(S-i)+\frac{1}{S-1}} dv_2 \right] \left. \right) \\
& - \left(\frac{-1}{S^{S+2}(S+1)} + \frac{1}{S^2} + \frac{(1 - \frac{1}{S(S-1)})^{S+1}}{S(S+1)} + \frac{1}{S^2(S-1)^2} \right) \left. \right] \\
& + \left[\left(\frac{1}{S} \left[\sum_{i=0}^S \frac{(-1 + (1 + \frac{1}{S(S-1)})^{i+1})}{i+1} \binom{S}{i} \left[\frac{-1}{[(S-1)(S-i)+1](S-1)^{S-i}} \right] \right) \right. \right. \\
& - \left. \left(\frac{(1 - \frac{1}{S(S-1)})^{S+1} - 1}{S(S+1)} \right) \right].
\end{aligned}$$

Alternating Auction

We now derive the expected profit to the supplier in the alternating auction when her private valuation in the last period is fixed but unknown. In equilibrium, the suppliers will use the optimal bid strategy derived in Proposition 3.4.2 (i.e., $b(v_2) = v_2 + \frac{B-3}{2(B+1)(S-1)}$).

The expected profit to the supplier when her private valuation in the last period v_1 is

unknown is as follows:

$$\begin{aligned}
Q_2(v_2, v_1) &= \int_{v_2}^1 (b(v_{(2)}) - v_2)(S-1)(1-F(v_{(2)}))^{S-2} f(v_{(2)}) dv_{(2)} \\
&\quad + \left(\frac{1}{S-1} \int_0^{v_2} \left(\int_0^1 w_1 B(B-1) F(w_1)^{B-2} (1-F(w_1)) f(w_1) dw_1 - v_1 \right) \right. \\
&\quad \left. * (S-1)(1-F(v_{(2)}))^{S-2} f(v_{(2)}) dv_{(2)} \right) \\
&= \int_{v_2}^1 \left(v_{(2)} + \frac{B-3}{2(B+1)(S-1)} - v_2 \right) (S-1)(1-F(v_{(2)}))^{S-2} f(v_{(2)}) dv_{(2)} \\
&\quad + \left(\frac{1}{S-1} \int_0^{v_2} \left(\frac{\frac{B-1}{B+1} - v_1}{S} \right) (S-1)(1-v_{(2)})^{S-2} dv_{(2)} \right) \\
&= - \left[(1-v_{(2)})^{S-1} \left(v_{(2)} + \frac{B-3}{2(B+1)(S-1)} - v_2 \right) \right]_{v_2}^1 + \int_{v_2}^1 (1-v_{(2)})^{S-1} dv_{(2)} \\
&\quad + \int_0^{v_2} \left(\frac{\frac{B-1}{B+1} - v_1}{S} \right) (1-v_{(2)})^{S-2} dv_{(2)} \\
&= (1-v_2)^{S-1} \left(\frac{B-3}{2(B+1)(S-1)} \right) + \frac{(1-v_2)^S}{S} + \frac{\frac{B-1}{B+1} - v_1}{S} \frac{(1-(1-v_2)^{S-1})}{S-1}.
\end{aligned}$$

We now derive the type that she will bid as for the alternating auction when v_1 is known. The expected profit is as follows:

$$\begin{aligned}
P_2(v_2, v_1) &= \int_r^1 (b(v_{(2)}) - v_2)(S-1)(1-F(v_{(2)}))^{S-2} f(v_{(2)}) dv_{(2)} \\
&\quad + \frac{1}{S-1} \int_0^r \left[\int_0^1 w B(B-1) F(w)^{B-2} (1-F(w)) f(w) dw - v_1 \right] \\
&\quad * (S-1)(1-F(v_{(2)}))^{S-2} f(v_{(2)}) dv_{(2)} \\
&= \int_r^1 (b(v_{(2)}) - v_2)(S-1)(1-v_{(2)})^{S-2} dv_{(2)} \\
&\quad + \frac{1}{S-1} \int_0^r \left(\frac{\frac{B-1}{B+1} - v_1}{S-1} \right) (S-1)(1-v_{(2)})^{S-2} dv_{(2)}.
\end{aligned}$$

We obtain the first order conditions using Leibnitz's rule with respect to the type, r , in the 2^{nd} -to-last period where r is a function of v_2 and v_1 , we have

$$\frac{dP_2(v_2, v_1)}{dr} = - \left[\left(r + \frac{B-3}{2(B+1)(S-1)} - v_2 \right) (S-1)(1-r)^{S-2} \right] + \left(\frac{\frac{B-1}{B+1} - v_1}{S-1} \right) (S-1)(1-r)^{S-2} \equiv 0.$$

We solve for the type r as follows:

$$\begin{aligned}
\left(r + \frac{B-3}{2(B+1)(S-1)} - v_2 \right) (S-1)(1-r)^{S-2} &= \left(\frac{\frac{B-1}{B+1} - v_1}{S-1} \right) (S-1)(1-r)^{S-2} \\
r + \frac{B-3}{2(B+1)(S-1)} - v_2 &= \frac{\frac{B-1}{B+1} - v_1}{S-1} \\
r = v_2 + \frac{\frac{B-1}{B+1} - v_1}{S-1} - \frac{B-3}{2(B+1)(S-1)}.
\end{aligned}$$

We define $g(v_2, v_1) = v_2 + \frac{\frac{B-1}{B+1} - v_1}{S-1} - \frac{B-3}{2(B+1)(S-1)}$. There are values of v_2 and v_1 that make $g(v_2, v_1) > 1$ or $g(v_2, v_1) < 0$ (i.e., $v_1 = 0$ and $v_2 = 1$ or $v_1 = 1$ and $v_2 = 0$). We now consider the expected profit for each of three cases: when $g(v_2, v_1) > 1$, when $g(v_2, v_1) < 0$, and when $0 < g(v_2, v_1) < 1$. We will use g in place of $g(v_2, v_1)$.

Limits of Integration We first consider the range of values of v_1 that correspond to $g(v_2, v_1) > 1$. The upper bound on v_1 is derived as follows:

$$\begin{aligned}
g(v_2, v_1) &> 1 \\
v_2 + \frac{\frac{B-1}{B+1} - v_1}{S-1} - \frac{B-3}{2(B+1)(S-1)} &> 1 \\
v_2 + \frac{\frac{B-1}{B+1} - v_1}{S-1} &> 1 + \frac{B-3}{2(B+1)(S-1)} \\
\frac{\frac{B-1}{B+1} - v_1}{S-1} &> 1 + \frac{B-3}{2(B+1)(S-1)} - v_2 \\
\frac{B-1}{B+1} - v_1 &> (S-1) + \frac{(B-3)(S-1)}{2(B+1)(S-1)} - v_2(S-1) \\
\frac{B-1}{B+1} - v_1 &> (1-v_2)(S-1) + \frac{(B-3)}{2(B+1)} \\
v_1 &< \frac{B-1}{B+1} - (1-v_2)(S-1) - \frac{(B-3)}{2(B+1)} \\
v_1 &< \frac{B-1}{B+1} - \frac{(B-3)}{2(B+1)} - (1-v_2)(S-1) \\
v_1 &< \frac{2B-2-B+3}{2(B+1)} - (1-v_2)(S-1) \\
v_1 &< \frac{B+1}{2(B+1)} - (1-v_2)(S-1) \\
v_1 &< \frac{1}{2} - (1-v_2)(S-1) \\
v_1 &< X
\end{aligned}$$

where $X = \frac{1}{2} - (1-v_2)(S-1)$. We now consider the values of v_2 for which $v_1 < X$ is not possible, given that v_1 is bounded by $[0, 1]$. Specifically, when $X \leq 0$, there is no value of v_1 for which $v_1 < X$ or for which $g > 1$. The inequality $X \leq 0$ holds when

$$\begin{aligned}
X &\leq 0 \\
\frac{1}{2} - (1-v_2)(S-1) &\leq 0 \\
\frac{1}{2} &\leq (1-v_2)(S-1) \\
\frac{1}{2(S-1)} &\leq 1-v_2 \\
v_2 &\leq 1 - \frac{1}{2(S-1)}
\end{aligned}$$

Therefore, when $v_2 \leq 1 - \frac{1}{2(S-1)}$ there is no v_1 for which $v_1 < X$. So when $v_2 > 1 - \frac{1}{2(S-1)}$,

we are able to consider the expected profit where $g > 1$.

We next consider the values of v_1 that correspond to $g(v_2, v_1) < 0$. If $v_1 \geq X$ then $g \leq 1$. The lower bound on v_1 is derived as follows:

$$\begin{aligned}
g(v_2, v_1) &< 0 \\
v_2 + \frac{\frac{B-1}{B+1} - v_1}{S-1} - \frac{B-3}{2(B+1)(S-1)} &< 0 \\
v_2 + \frac{\frac{B-1}{B+1} - v_1}{S-1} &< \frac{B-3}{2(B+1)(S-1)} \\
\frac{\frac{B-1}{B+1} - v_1}{S-1} &< \frac{B-3}{2(B+1)(S-1)} - v_2 \\
\frac{B-1}{B+1} - v_1 &< \frac{(B-3)(S-1)}{2(B+1)(S-1)} - v_2(S-1) \\
\frac{B-1}{B+1} - v_1 &< \frac{(B-3)}{2(B+1)} - v_2(S-1) \\
v_1 &> \frac{B-1}{B+1} - \frac{(B-3)}{2(B+1)} + v_2(S-1) \\
v_1 &> \frac{2B-2-B+3}{2(B+1)} + v_2(S-1) \\
v_1 &> \frac{B+1}{2(B+1)} + v_2(S-1) \\
v_1 &> \frac{1}{2} + v_2(S-1) \\
v_1 &> Y
\end{aligned}$$

where $Y = \frac{1}{2} + v_2(S-1)$.

We now consider the values of v_2 for which $v_1 > Y$ is not possible, given that v_1 is bounded by $[0, 1]$. Specifically, when $Y \geq 1$, there is no value of v_1 for which $v_1 > Y$ or for which $g < 0$. The inequality $Y \geq 1$ holds when

$$\begin{aligned}
Y &\geq 1 \\
\frac{1}{2} + v_2(S-1) &\geq 1 \\
v_2(S-1) &\geq 1 - \frac{1}{2} \\
v_2 &\geq \frac{1}{2(S-1)}
\end{aligned}$$

Therefore, when $v_2 \geq \frac{1}{2(S-1)}$, there is no v_1 for which $v_1 > Y$. So when $v_2 < \frac{1}{2(S-1)}$ we are able to consider the expected profit when $g < 0$.

When we compare the bounds on v_2 we have

$$\begin{aligned}
& 1 - \frac{1}{2(S-1)} - \frac{1}{2(S-1)} \\
&= \frac{2S-3}{2(S-1)} - \frac{1}{2(S-1)} \\
&= \frac{2S-4}{2(S-1)} \\
&= \frac{2(S-2)}{2(S-1)} \\
&= \frac{S-2}{S-1} > 0 \text{ when } S > 2
\end{aligned}$$

Therefore, $1 - \frac{1}{2(S-1)} > \frac{1}{2(S-1)}$ and the bounds on v_2 for $g < 0$ and $g > 1$ do not intersect.

We also consider the limits of integration when $0 \leq g \leq 1$. These limits are $X \leq v_1 \leq Y$ with the corresponding bounds for v_2 as $\frac{1}{2(S-1)} \leq v_2 \leq \frac{2S-3}{2(S-1)}$.

With these limits of integration, we obtain the expected value of information to the supplier who knows what her private valuation will be in the last period for the cases when $g(v_2, v_1) > 1$, $g(v_2, v_1) < 0$, and $0 < g(v_2, v_1) < 1$.

Case 1: In this case, $g(v_2, v_1) > 1$ so the strategic supplier will lose the first auction and have the opportunity to receive an expected profit from initiating a forward auction in the last period. If $v_2 > \frac{2S-3}{2(S-1)}$, the expected value of learning v_1 in the 2^{nd} -to-last period after v_2 is known ($EVI_A(v_2)$) is as follows:

$$\begin{aligned}
EVI_A(v_2) &= \int_0^X \left(\frac{\frac{B-1}{B+1} - v_1}{S-1} \right) f(v_1) dv_1 \\
&+ \int_X^1 \left(\frac{\frac{B-1}{B+1} - v_1}{S-1} + \frac{(1 - v_2 - \frac{\frac{B-1}{B+1} - v_1}{S-1} + \frac{B-3}{2(B+1)(S-1)})^S}{S} \right) f(v_1) dv_1 \\
&- \int_0^1 Q_2(v_2, v_1) f(v_1) dv_1.
\end{aligned}$$

where the first integrand is the expected profit in the last period and follows from Proposition 3.4.3 for P_1 . The third integrand is the expected profit to the supplier when she does not know v_1 until the last period so it is integrated over all possible values of v_1 . The second integrand is the expected profit when $0 \leq g \leq 1$ because although $v_2 > \frac{2S-3}{2(S-1)}$, it is possible that $v_1 > X$. When this occurs, the supplier may either win as a bidder or

initiate as an auctioneer. The integrand is derived as follows:

$$\begin{aligned}
& \int_g^1 (b(v_{(2)}) - v_2)(S-1)(1-F(v_{(2)}))^{S-2} f(v_{(2)}) dv_{(2)} \\
& + \int_0^g \left[\frac{1}{S-1} \int_0^1 wB(B-1)F(w)^{B-2}(1-F(w))f(w)dw - v_1 \right] \\
& * (S-1)(1-F(v_{(2)}))^{S-2} f(v_{(2)}) dv_{(2)} \Big] dv_1 \\
= & \int_g^1 (v_{(2)} + \frac{B-3}{2(B+1)(S-1)} - v_2)(S-1)(1-v_{(2)})^{S-2} dv_{(2)} \\
& + \int_0^g \left[\frac{1}{S-1} \int_0^1 B(B-1)(w^{B-1} - w^B)dw - v_1 \right] (S-1)(1-v_{(2)})^{S-2} dv_{(2)} \\
= & - \left[(1-v_{(2)})^{S-1} (v_{(2)} + \frac{B-3}{2(B+1)(S-1)} - v_2) \right]_g^1 + \int_g^1 (1-v_{(2)})^{S-1} dv_{(2)} \\
& + \int_0^g \left(\frac{B-1}{B+1} - v_1 \right) (1-v_{(2)})^{S-2} dv_{(2)} \\
= & \left(\frac{B-1}{B+1} - v_1 \right) \left(\frac{(1-g)^{S-1}}{S-1} \right) + \frac{(1-g)^S}{S} + \left(\frac{B-1}{B+1} - v_1 \right) \frac{(1-v_{(2)})^{S-1}}{S-1} \Big|_0^g \\
= & \left(\frac{B-1}{B+1} - v_1 \right) \left(\frac{(1-g)^{S-1}}{S-1} \right) + \frac{(1-g)^S}{S} + \left(\frac{B-1}{B+1} - v_1 \right) \left(\frac{1-(1-g)^{S-1}}{S-1} \right) \\
= & \frac{\frac{B-1}{B+1} - v_1}{S-1} + \frac{(1-g)^S}{S} \\
= & \frac{\frac{B-1}{B+1} - v_1}{S-1} + \frac{\left(1 - v_2 - \frac{\frac{B-1}{B+1} - v_1}{S-1} + \frac{B-3}{2(B+1)(S-1)} \right)^S}{S}.
\end{aligned} \tag{45}$$

$EVI_A(v_2)$ is simplified as follows:

$$\begin{aligned}
EVI_A(v_2) &= \int_0^X \left(\frac{\frac{B-1}{B+1} - v_1}{S-1} \right) f(v_1) dv_1 \\
&+ \int_X^1 \left(\frac{\frac{B-1}{B+1} - v_1}{S-1} + \frac{(1-v_2 - \frac{\frac{B-1}{B+1} - v_1}{S-1} + \frac{B-3}{2(B+1)(S-1)})^S}{S} \right) f(v_1) dv_1 \\
&- \int_0^1 \left[(1-v_2)^{S-1} \left(\frac{B-3}{2(B+1)(S-1)} \right) + \frac{(1-v_2)^S}{S} + \frac{\frac{B-1}{B+1} - v_1}{S} \frac{(1-(1-v_2)^{S-1})}{S-1} \right] f(v_1) dv_1 \\
&= \left(\frac{\frac{B-1}{B+1} v_1 - \frac{v_1^2}{2}}{S-1} \right) \Big|_0^X \\
&+ \left(\frac{\frac{B-1}{B+1} v_1 - \frac{v_1^2}{2}}{S-1} + \frac{(1-v_2 - \frac{\frac{B-1}{B+1} - v_1}{S-1} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \right) \Big|_X^1 \\
&- \left[(1-v_2)^{S-1} v_1 \left(\frac{B-3}{2(B+1)(S-1)} \right) + \frac{(1-v_2)^S}{S} v_1 + \frac{\frac{B-1}{B+1} v_1 - \frac{v_1^2}{2}}{S} \frac{(1-(1-v_2)^{S-1})}{S-1} \right] \Big|_0^1 \\
&= \left(\frac{\frac{B-1}{B+1} X - \frac{X^2}{2}}{S-1} \right) \\
&+ \left(\frac{\frac{B-1}{B+1} - \frac{1}{2}}{S-1} + \frac{(1-v_2 - \frac{\frac{B-1}{B+1} - 1}{S-1} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \right. \\
&- \left. \left(\frac{\frac{B-1}{B+1} X - \frac{X^2}{2}}{S-1} + \frac{(1-v_2 - \frac{\frac{B-1}{B+1} - X}{S-1} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \right) \right) \\
&- \left[(1-v_2)^{S-1} \left(\frac{B-3}{2(B+1)(S-1)} \right) + \frac{(1-v_2)^S}{S} + \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S} \frac{(1-(1-v_2)^{S-1})}{S-1} \right].
\end{aligned}$$

Case 2: In this case, $g(v_2, v_1) < 0$ so the strategic supplier will win the first auction and not participate in the last period. If $v_2 < \frac{1}{2(S-1)}$, the expected value of learning v_1 in the 2^{nd} -to-last period after v_2 is known ($EVI_A(v_2)$) is as follows:

$$\begin{aligned}
EVI_A(v_2) &= \int_Y^1 \left(\frac{B-1}{2(B+1)(S-1)} - v_2 + \frac{1}{S} \right) f(v_1) dv_1 \\
&+ \int_0^Y \left(\frac{\frac{B-1}{B+1} - v_1}{S-1} + \frac{(1-v_2 - \frac{\frac{B-1}{B+1} - v_1}{S-1} + \frac{B-3}{2(B+1)(S-1)})^S}{S} \right) f(v_1) dv_1 \\
&- \int_0^1 Q_2(v_2, v_1) f(v_1) dv_1.
\end{aligned}$$

The second integrand is the expected profit when $0 \leq g \leq 1$ because although $v_2 < \frac{1}{2(S-1)}$, it is possible that $v_1 < Y$. When this occurs, the supplier may either win as a bidder or initiate as an auctioneer. The derivation that yields this integrand is given in (45). The third integrand is the expected profit to the supplier when she does not know v_1 until the last period so it is integrated over all possible values of v_1 . The first integrand is

the expected profit in the 2^{nd} -to-last period and is derived as follow:

$$\begin{aligned}
& \int_0^1 (b(v_{(2)}) - v_2)(S-1)(1-F(v_{(2)}))^{S-2} f(v_{(2)}) dv_{(2)} \\
&= \int_0^1 \left(v_{(2)} + \frac{B-3}{2(B+1)(S-1)} - v_2 \right) (S-1)(1-v_{(2)})^{S-2} \\
&= - \left[(1-v_{(2)})^{S-1} \left(v_{(2)} + \frac{B-3}{2(B+1)(S-1)} - v_2 \right) \right]_0^1 + \int_0^1 (1-v_{(2)})^{S-1} dv_{(2)} \\
&= \frac{B-3}{2(B+1)(S-1)} - v_2 + \frac{1}{S}
\end{aligned}$$

$EVI_A(v_2)$ is simplified as follows:

$$\begin{aligned}
EVI_A(v_2) &= \int_Y^1 \left(\frac{B-1}{2(B+1)(S-1)} - v_2 + \frac{1}{S} \right) f(v_1) dv_1 \\
&+ \int_0^Y \left(\frac{\frac{B-1}{B+1} - v_1}{S-1} + \frac{(1-v_2 - \frac{\frac{B-1}{B+1} - v_1}{S-1} + \frac{B-3}{2(B+1)(S-1)})^S}{S} \right) f(v_1) dv_1 \\
&- \int_0^1 Q_2(v_2, v_1) f(v_1) dv_1 \\
&= \left(\frac{B-1}{2(B+1)(S-1)} v_1 - v_2 v_1 + \frac{v_1}{S} \right) \Big|_Y^1 \\
&+ \left(\frac{\frac{B-1}{B+1} v_1 - \frac{v_1^2}{2}}{S-1} + \frac{(1-v_2 - \frac{\frac{B-1}{B+1} - v_1}{S-1} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \right) \Big|_0^Y \\
&- \left[(1-v_2)^{S-1} v_1 \left(\frac{B-3}{2(B+1)(S-1)} \right) + \frac{(1-v_2)^S}{S} v_1 + \frac{\frac{B-1}{B+1} v_1 - \frac{v_1^2}{2}}{S} \frac{(1-(1-v_2)^{S-1})}{S-1} \right] \Big|_0^1 \\
&= \left(\frac{B-1}{2(B+1)(S-1)} - v_2 + \frac{1}{S} \right) - \left(\frac{B-1}{2(B+1)(S-1)} Y - v_2 Y + \frac{Y}{S} \right) \\
&+ \left(\frac{\frac{B-1}{B+1} Y - \frac{Y^2}{2}}{S-1} + \frac{(1-v_2 - \frac{\frac{B-1}{B+1} - Y}{S-1} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \right) \\
&- \left(\frac{(1-v_2 - \frac{B-1}{(B+1)(S-1)} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \right) \\
&- \left[(1-v_2)^{S-1} \left(\frac{B-3}{2(B+1)(S-1)} \right) + \frac{(1-v_2)^S}{S} + \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S} \frac{(1-(1-v_2)^{S-1})}{S-1} \right].
\end{aligned}$$

Case 3: In this case, $0 < g(v_2, v_1) < 1$, the strategic supplier has a positive probability of either winning the first auction or initiating the second auction. If $\frac{1}{2(S-1)} \leq v_2 \leq 1 - \frac{1}{2(S-1)}$, the expected value of learning v_1 in the 2^{nd} -to-last period after v_2 is known ($EVI_A(v_2)$) is as follows:

$$\begin{aligned}
EVI_A(v_2) &= \int_0^1 \left(\frac{\frac{B-1}{B+1} - v_1}{S-1} + \frac{(1-v_2 - \frac{\frac{B-1}{B+1} - v_1}{S-1} + \frac{B-3}{2(B+1)(S-1)})^S}{S} \right) f(v_1) dv_1 \\
&- \int_0^1 Q_2(v_2, v_1) f(v_1) dv_1.
\end{aligned}$$

The first integrand is the expected profit when $0 \leq g \leq 1$ with $\frac{1}{2(S-1)} \leq v_2 \leq 1 - \frac{1}{2(S-1)}$. In this range, it is possible that $0 \leq v_1 \leq 1$. When this occurs, the supplier may either win

as a bidder or initiate as an auctioneer. The derivation that yields this integrand is given in (45).

$EVI_A(v_2)$ is simplified as follows:

$$\begin{aligned}
EVI_A(v_2) &= \int_0^1 \left(\frac{\frac{B-1}{B+1} - v_1}{S-1} + \frac{(1-v_2 - \frac{\frac{B-1}{B+1} - v_1}{S-1} + \frac{B-3}{2(B+1)(S-1)})^S}{S} \right) f(v_1) dv_1 \\
&\quad - \int_0^1 Q_2(v_2, v_1) f(v_1) dv_1 \\
&= \left(\frac{\frac{B-1}{B+1} v_1 - \frac{v_1^2}{2}}{S-1} + \frac{(1-v_2 - \frac{\frac{B-1}{B+1} - v_1}{S-1} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \right) \Big|_0^1 \\
&\quad - \left[(1-v_2)^{S-1} v_1 \left(\frac{B-3}{2(B+1)(S-1)} \right) + \frac{(1-v_2)^S}{S} v_1 + \frac{\frac{B-1}{B+1} v_1 - \frac{v_1^2}{2}}{S} \frac{(1-(1-v_2)^{S-1})}{S-1} \right] \Big|_0^1 \\
&= \left(\frac{\frac{B-1}{B+1} - \frac{1}{2}}{S-1} + \frac{(1-v_2 - \frac{\frac{B-1}{B+1} - 1}{S-1} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \right) \\
&\quad - \left(\frac{(1-v_2 - \frac{B-1}{(B+1)(S-1)} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \right) \\
&\quad - \left[(1-v_2)^{S-1} \left(\frac{B-3}{2(B+1)(S-1)} \right) + \frac{(1-v_2)^S}{S} + \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S} \frac{(1-(1-v_2)^{S-1})}{S-1} \right].
\end{aligned}$$

To determine the amount that a supplier would be willing to pay to learn v_1 in the 2^{nd} -to-last period before v_2 is known, (EVI_B) , we integrate the expected value of information

in Cases 1-3 with respect to v_2 as well then sum them to obtain the following:

$$\begin{aligned}
EVI_B &= \int_{1-\frac{1}{2(S-1)}}^1 \left[\int_0^X \left(\frac{\frac{B-1}{B+1} - v_1}{S-1} \right) f(v_1) dv_1 \right. \\
&\quad + \int_X^1 \left(\frac{\frac{B-1}{B+1} - v_1}{S-1} + \frac{(1-v_2 - \frac{\frac{B-1}{B+1} - v_1}{S-1} + \frac{B-3}{2(B+1)(S-1)})^S}{S} \right) f(v_1) dv_1 \\
&\quad - \int_0^1 Q_2(v_2, v_1) f(v_1) dv_1 \Big] f(v_2) dv_2 + \int_0^{\frac{1}{2(S-1)}} \left[\int_Y^1 \left(\frac{B-1}{2(B+1)(S-1)} - v_2 + \frac{1}{S} \right) f(v_1) dv_1 \right. \\
&\quad + \int_0^Y \left(\frac{\frac{B-1}{B+1} - v_1}{S-1} + \frac{(1-v_2 - \frac{\frac{B-1}{B+1} - v_1}{S-1} + \frac{B-3}{2(B+1)(S-1)})^S}{S} \right) f(v_1) dv_1 \\
&\quad - \int_0^1 Q_2(v_2, v_1) f(v_1) dv_1 \Big] f(v_2) dv_2 \\
&\quad + \int_{\frac{1}{2(S-1)}}^{1-\frac{1}{2(S-1)}} \left[\int_0^1 \left(\frac{\frac{B-1}{B+1} - v_1}{S-1} + \frac{(1-v_2 - \frac{\frac{B-1}{B+1} - v_1}{S-1} + \frac{B-3}{2(B+1)(S-1)})^S}{S} \right) f(v_1) dv_1 \right. \\
&\quad - \int_0^1 Q_2(v_2, v_1) f(v_1) dv_1 \Big] f(v_2) dv_2 \\
&= \int_{1-\frac{1}{2(S-1)}}^1 \left[\left(\frac{\frac{B-1}{B+1}X - \frac{X^2}{2}}{S-1} \right) \right. \\
&\quad + \left(\frac{\frac{B-1}{B+1} - \frac{1}{2}}{S-1} + \frac{(1-v_2 - \frac{\frac{B-1}{B+1} - 1}{S-1} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \right) \\
&\quad - \left(\frac{\frac{B-1}{B+1}X - \frac{X^2}{2}}{S-1} + \frac{(1-v_2 - \frac{\frac{B-1}{B+1} - X}{S-1} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \right) \\
&\quad - \left((1-v_2)^{S-1} \left(\frac{B-3}{2(B+1)(S-1)} \right) + \frac{(1-v_2)^S}{S} + \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S} \frac{(1-(1-v_2)^{S-1})}{S-1} \right) \Big] f(v_2) dv_2 \\
&\quad + \int_0^{\frac{1}{2(S-1)}} \left[\left(\frac{B-1}{2(B+1)(S-1)} - v_2 + \frac{1}{S} \right) - \left(\frac{B-1}{2(B+1)(S-1)}Y - v_2Y + \frac{Y}{S} \right) \right. \\
&\quad + \left(\frac{\frac{B-1}{B+1}Y - \frac{Y^2}{2}}{S-1} + \frac{(1-v_2 - \frac{\frac{B-1}{B+1} - Y}{S-1} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \right) \\
&\quad - \left(\frac{(1-v_2 - \frac{B-1}{(B+1)(S-1)} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \right) \\
&\quad - \left((1-v_2)^{S-1} \left(\frac{B-3}{2(B+1)(S-1)} \right) + \frac{(1-v_2)^S}{S} + \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S} \frac{(1-(1-v_2)^{S-1})}{S-1} \right) \Big] f(v_2) dv_2 \\
&\quad + \int_{\frac{1}{2(S-1)}}^{1-\frac{1}{2(S-1)}} \left[\left(\frac{\frac{B-1}{B+1} - \frac{1}{2}}{S-1} + \frac{(1-v_2 - \frac{\frac{B-1}{B+1} - 1}{S-1} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \right) \right. \\
&\quad - \left(\frac{(1-v_2 - \frac{B-1}{(B+1)(S-1)} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \right) \\
&\quad - \left((1-v_2)^{S-1} \left(\frac{B-3}{2(B+1)(S-1)} \right) + \frac{(1-v_2)^S}{S} + \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S} \frac{(1-(1-v_2)^{S-1})}{S-1} \right) \Big] f(v_2) dv_2
\end{aligned}$$

We continue after substituting the values of X and Y where where $X = \frac{1}{2} - (1-v_2)(S-1)$

and $Y = \frac{1}{2} + v_2(S-1)$.

$$\begin{aligned}
EVI_B = & \int_{1-\frac{1}{2(S-1)}}^1 \left[\left(\frac{\frac{B-1}{B+1} - \frac{1}{2}}{S-1} + \frac{(1-v_2 - \frac{\frac{B-1}{B+1}-1}{S-1} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \right) \right. \\
& + \frac{(1-v_2 - \frac{\frac{B-1}{B+1} - (\frac{1}{2} - (1-v_2)(S-1))}{S-1} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \Big) \\
& - \left((1-v_2)^{S-1} \left(\frac{B-3}{2(B+1)(S-1)} \right) + \frac{(1-v_2)^S}{S} + \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S} \frac{(1-(1-v_2)^{S-1})}{S-1} \right) \Big] f(v_2) dv_2 \\
& + \int_0^{\frac{1}{2(S-1)}} \left[\left(\frac{B-1}{2(B+1)(S-1)} - v_2 + \frac{1}{S} \right) \right. \\
& - \left(\frac{B-1}{2(B+1)(S-1)} \left(\frac{1}{2} + v_2(S-1) \right) - v_2 \left(\frac{1}{2} + v_2(S-1) \right) + \frac{\frac{1}{2} + v_2(S-1)}{S} \right) \\
& + \frac{\frac{B-1}{B+1} (\frac{1}{2} + v_2(S-1)) - \frac{(\frac{1}{2} + v_2(S-1))^2}{2}}{S-1} \\
& + \frac{(1-v_2 - \frac{\frac{B-1}{B+1} - (\frac{1}{2} + v_2(S-1))}{S-1} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \\
& - \left(\frac{(1-v_2 - \frac{B-1}{(B+1)(S-1)} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \right) \\
& - \left((1-v_2)^{S-1} \left(\frac{B-3}{2(B+1)(S-1)} \right) + \frac{(1-v_2)^S}{S} + \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S} \frac{(1-(1-v_2)^{S-1})}{S-1} \right) \Big] f(v_2) dv_2 \\
& + \int_{\frac{1}{2(S-1)}}^{1-\frac{1}{2(S-1)}} \left[\left(\frac{\frac{B-1}{B+1} - \frac{1}{2}}{S-1} + \frac{(1-v_2 - \frac{\frac{B-1}{B+1}-1}{S-1} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \right) \right. \\
& - \left(\frac{(1-v_2 - \frac{B-1}{(B+1)(S-1)} + \frac{B-3}{2(B+1)(S-1)})^{S+1}}{S(S+1)} (S-1) \right) \\
& - \left((1-v_2)^{S-1} \left(\frac{B-3}{2(B+1)(S-1)} \right) + \frac{(1-v_2)^S}{S} + \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S} \frac{(1-(1-v_2)^{S-1})}{S-1} \right) \Big] f(v_2) dv_2
\end{aligned}$$

We integrate over the respective ranges of v_2 as follows:

$$\begin{aligned}
EVI_B = & \left[\left(\left(\frac{B-1}{B+1} - \frac{1}{2} \right) v_2 - \frac{(1-v_2 - \frac{B-1}{B+1} - \frac{1}{S-1} + \frac{B-3}{2(B+1)(S-1)})^{S+2}}{S(S+1)(S+2)} (S-1) \right) \Big|_{1-\frac{1}{2(S-1)}}^1 \right. \\
& + \left(\frac{-(2(1-v_2))^{S+2}}{S(S+1)(S+2)} (S-1) \right) \Big|_{1-\frac{1}{2(S-1)}}^1 \\
& - \left(\frac{-(1-v_2)^S}{S} \left(\frac{B-3}{2(B+1)(S-1)} \right) + \frac{-(1-v_2)^{S+1}}{S(S+1)} + \frac{B-1}{B+1} - \frac{1}{2} \frac{(v_2 + (1-v_2)^S)}{S(S-1)} \right) \Big|_{1-\frac{1}{2(S-1)}}^1 \\
& + \left[\left(\frac{(B-1)v_2}{2(B+1)(S-1)} - \frac{v_2^2}{2} + \frac{v_2}{S} \right) \Big|_0^{\frac{1}{2(S-1)}} \right. \\
& - \left(\frac{B-1}{2(B+1)(S-1)} \left(\frac{v_2}{2} + \frac{v_2^2}{2} (S-1) \right) - \frac{v_2^2}{2} - \frac{v_2^3}{3} (S-1) + \frac{v_2}{2} + \frac{v_2^2}{2} (S-1) \right) \Big|_0^{\frac{1}{2(S-1)}} \\
& + \left(\frac{B-1}{B+1} \left(\frac{v_2}{2} + \frac{v_2^2}{2} (S-1) \right) - \frac{(\frac{1}{2} + v_2(S-1))^3}{6(S-1)} + \frac{-(1-2v_2)^{S+2}}{2S(S+1)(S+2)} (S-1) \right) \Big|_0^{\frac{1}{2(S-1)}} \\
& - \left(\frac{-(1-v_2 - \frac{B-1}{(B+1)(S-1)} + \frac{B-3}{2(B+1)(S-1)})^{S+2}}{S(S+1)(S+2)} (S-1) \right) \Big|_0^{\frac{1}{2(S-1)}} \\
& - \left(\frac{-(1-v_2)^S}{S} \left(\frac{B-3}{2(B+1)(S-1)} \right) + \frac{-(1-v_2)^{S+1}}{S(S+1)} + \frac{B-1}{B+1} - \frac{1}{2} \frac{(v_2 + (1-v_2)^S)}{S(S-1)} \right) \Big|_{1-\frac{1}{2(S-1)}}^1 \\
& + \left[\left(\left(\frac{B-1}{B+1} - \frac{1}{2} \right) v_2 + \frac{-(1-v_2 - \frac{B-1}{B+1} - \frac{1}{S-1} + \frac{B-3}{2(B+1)(S-1)})^{S+2}}{S(S+1)(S+2)} (S-1) \right) \Big|_{\frac{1}{2(S-1)}}^{1-\frac{1}{2(S-1)}} \right. \\
& - \left(\frac{-(1-v_2 - \frac{B-1}{(B+1)(S-1)} + \frac{B-3}{2(B+1)(S-1)})^{S+2}}{S(S+1)(S+2)} (S-1) \right) \Big|_{\frac{1}{2(S-1)}}^{1-\frac{1}{2(S-1)}} \\
& \left. - \left(\frac{-(1-v_2)^S}{S} \left(\frac{B-3}{2(B+1)(S-1)} \right) + \frac{-(1-v_2)^{S+1}}{S(S+1)} + \frac{B-1}{B+1} - \frac{1}{2} \frac{(v_2 + (1-v_2)^S)}{S(S-1)} \right) \Big|_{\frac{1}{2(S-1)}}^{1-\frac{1}{2(S-1)}} \right].
\end{aligned}$$

We simplify as follows:

$$\begin{aligned}
EVI_B = & \left[\left(\left(\frac{B-1}{B+1} - \frac{1}{2} \right) - \frac{(-\frac{B-1}{B+1}-1 + \frac{B-3}{2(B+1)(S-1)})^{S+2}}{S(S+1)(S+2)}(S-1) \right) \right. \\
& - \left(\left(\frac{B-1}{B+1} - \frac{1}{2} \right) \left(\frac{2S-3}{2(S-1)} \right) + \frac{(1 - (\frac{2S-3}{2(S-1)}) - \frac{B-1}{B+1}-1 + \frac{B-3}{2(B+1)(S-1)})^{S+2}}{S(S+1)(S+2)}(S-1) \right) \\
& + \left(\frac{1}{S(S+1)(S+2)(S-1)^{S+2}}(S-1) \right) - \left(\frac{\frac{B-1}{B+1} - \frac{1}{2}}{S} \frac{1}{S(S-1)} \right) \\
& + \left(\frac{-1}{S(2(S-1))^S} \left(\frac{B-3}{2(B+1)(S-1)} \right) + \frac{-1}{S(S+1)(2(S-1))^{S+1}} \right. \\
& \left. + \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S} \frac{(\frac{2S-3}{2(S-1)} + \frac{1}{(2(S-1))^S})}{S(S-1)} \right) \Big] \\
& + \left[\left(\frac{(B-1)}{4(B+1)(S-1)^2} - \frac{1}{2} + \frac{1}{2S(S-1)} \right) \right. \\
& - \left(\frac{B-1}{2(B+1)(S-1)} \left(\frac{1}{4(S-1)} + \frac{1}{8(S-1)} \right) - \frac{1}{8(S-1)^2} - \frac{1}{24(S-1)^2} + \frac{\frac{1}{4(S-1)} + \frac{1}{8(S-1)}}{S} \right) \\
& + \left(\frac{\frac{B-1}{B+1}(\frac{1}{4(S-1)} + \frac{1}{8(S-1)}) - \frac{1}{6(S-1)}}{S-1} + \frac{-(\frac{S-2}{S-1})^{S+2}}{2S(S+1)(S+2)}(S-1) \right. \\
& + \frac{1}{48(S-1)^2} + \frac{1}{2S(S+1)(S+2)}(S-1) \Big) \\
& + \frac{(\frac{2S-3}{2(S-1)} - \frac{B-1}{(B+1)(S-1)} + \frac{B-3}{2(B+1)(S-1)})^{S+2}}{S(S+1)(S+2)}(S-1) \\
& - \left(\frac{(1 - \frac{B-1}{(B+1)(S-1)} + \frac{B-3}{2(B+1)(S-1)})^{S+2}}{S(S+1)(S+2)}(S-1) \right) \\
& - \left(\frac{\frac{B-1}{B+1} - \frac{1}{2}}{S} \frac{1}{S(S-1)} \right) \\
& + \frac{-1}{S(2(S-1))^S} \left(\frac{B-3}{2(B+1)(S-1)} \right) + \frac{-1}{S(S+1)(2(S-1))^{S+1}} \\
& \left. + \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S} \frac{(1 - \frac{1}{2(S-1)} + \frac{1}{(2(S-1))^S})}{S(S-1)} \right] \\
& + \left[\left(\left(\frac{B-1}{B+1} - \frac{1}{2} \right) \left(\frac{2S-3}{2(S-1)} \right) + \frac{-(\frac{1}{2(S-1)} - \frac{B-1}{B+1}-1 + \frac{B-3}{2(B+1)(S-1)})^{S+2}}{S(S+1)(S+2)}(S-1) \right) \right. \\
& - \left[\left(\left(\frac{B-1}{B+1} - \frac{1}{2} \right) \left(\frac{1}{2(S-1)} \right) + \frac{(\frac{2S-3}{2(S-1)} - \frac{B-1}{B+1}-1 + \frac{B-3}{2(B+1)(S-1)})^{S+2}}{S(S+1)(S+2)}(S-1) \right) \right. \\
& + \left(\frac{(\frac{1}{2(S-1)} - \frac{B-1}{(B+1)(S-1)} + \frac{B-3}{2(B+1)(S-1)})^{S+2}}{S(S+1)(S+2)}(S-1) \right) \\
& - \left(\frac{(\frac{2S-3}{2(S-1)} - \frac{B-1}{(B+1)(S-1)} + \frac{B-3}{2(B+1)(S-1)})^{S+2}}{S(S+1)(S+2)}(S-1) \right) \\
& - \left(\frac{-(\frac{1}{2(S-1)})^S}{S} \left(\frac{B-3}{2(B+1)(S-1)} \right) + \frac{-(\frac{1}{2(S-1)})^{S+1}}{S(S+1)} + \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S} \frac{(\frac{2S-3}{2(S-1)} + (\frac{1}{2(S-1)})^S)}{S(S-1)} \right) \\
& \left. + \left(\frac{-(\frac{2S-3}{2(S-1)})^S}{S} \left(\frac{B-3}{2(B+1)(S-1)} \right) + \frac{-(\frac{2S-3}{2(S-1)})^{S+1}}{S(S+1)} + \frac{\frac{B-1}{B+1} - \frac{1}{2}}{S} \frac{(\frac{1}{2(S-1)} + (\frac{2S-3}{2(S-1)})^S)}{S(S-1)} \right) \right].
\end{aligned}$$

■

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VITA

Kendra Cherie Taylor was born in Los Angeles, California and raised in Salt Lake City, Utah. Her parents emphasized the importance of education early in her childhood and enrolled her in Montessori school at the age of two. Because of their tireless efforts to encourage scholastic effort and balance, she excelled in academic, sports, artistic and leadership activities.

Her preparation accompanied by God's grace afforded her a full scholarship to attend her first choice, Hampton University, a historically Black university in Hampton, Virginia. The program administering her scholarship had the goal of preparing Kendra to undertake doctoral studies upon graduation. As a result, she was involved in research activities throughout her undergraduate career. In 1999, she graduated Summa Cum Laude from Hampton with a B.S. degree in Mathematics.

Upon completing her undergraduate studies, Kendra had done enough research to determine the area in which she would pursue her Ph.D. and decided to do so at Georgia Tech. Her experiences at Hampton University confirmed her desire to become an educator, and hence seek the terminal degree.

During her tenure as a PhD student, Kendra was the recipient of numerous fellowship awards; including Packard Foundation, Center for Paper Business and Industry Studies, Student-Teacher Enhancement Partnership, FACES fellowship program, and SREB-AGEP Doctoral Colloquium. She also coordinated an ISyE women's breakfast for the female Ph.D. students in her department.

In 2001, Kendra earned her M.S. degree in Industrial Engineering, and in August, 2005, earned a Ph.D. in the same field. Kendra has a natural love for teaching, and a strong desire to become an educator. Accordingly, her post-graduation objective is to pursue a career in academia after gaining some industry experience. Besides her research and teaching interests, Kendra also enjoys travel, track and field, and community service.